Eur. Phys. J. C 13, 99–115 (2000)
Digital Object Identifier (DOI) 10.1007/s100520000250

THE EUROPEAN PHYSICAL JOURNAL C

Higgs bosons in the left–right model

G.G. Boyarkina[1], O.M. Boyarkin[1,a], A.N. Senko[2]

[1] Department of Physics, Minsk State Pedagogical University, Soviet Street 18, Minsk, 220050, Belarus
[2] Department of Physics, Grodno State University, Ogheshko Street 22, Grodno, 230023, Belarus

Received: 29 January 1999 / Revised version: 20 September 1999 /
Published online: 3 February 2000 – © Springer-Verlag 2000

Abstract. The model with the $SU(2)_{\rm L} \times SU(2)_{\rm R} \times U(1)_{B-L}$ gauge group, containing one bidoublet and two triplets in the Higgs sector, is considered. The link between the constants determining the physical Higgs boson interactions and the neutrino oscillation parameters is found. It is shown that the observation of the ultrahigh-energy neutrinos with the help of the processes $e^-\nu_e \to W^-Z, e^-\nu_e \to \mu^-\nu_\mu$, gives us information on the singly charged Higgs bosons. The processes of the doubly charged Higgs bosons production, $e^-\mu^- \to \Delta_1^{(--)}\gamma, e^-\mu^- \to \Delta_1^{(--)}Z$, are investigated. From the point of view of detecting the neutral Higgs bosons the process of the electron–muon recharge $e^-\mu^+ \to e^+\mu^-$ is studied.

1 Introduction

For the unified gauge field theories such basic properties as unitarity, renormalizability and endowing the fundamental particles with masses are strongly connected with the existence of the Higgs bosons. The observation of one or several Higgs bosons would establish the Higgs mechanism as the physical basis of the symmetry breaking. However, even for the standard model (SM) the Higgs boson sector is a crucial part of the theory still escaping direct experimental verification. Direct experiments searching for the SM Higgs at the LEP II e^+e^- collider have placed a lower limit of 89.3 GeV on its mass.

It is quite possible that the actual scalar sector in nature has more than one doublet of Higgs bosons or has Higgs bosons in other multiplets. This is expected in many theories that go beyond the SM. One such extension of the SM is provided by supersymmetry and the desire to tackle the hierarchy problem [1]. Another attractive motivation for extending the Higgs sector is to generate the small masses of the left-handed neutrinos whose existence is hinted at by present solar and atmospheric neutrinos data, as well as cosmological observations of the large scale structure and the possible explaining of the hot dark matter picture in the universe. Amongst the extensions of the SM having a massive neutrino the left–right models (LRMs) based on the gauge group $SU(2)_{\rm L} \times SU(2)_{\rm R} \times U(1)_{B-L}$ [2,3] are of special interest because the LRMs also offer a dynamical solution to the problem of parity violation of the weak interaction. The symmetric LRMs, because of their invariance under a discrete LR symmetry, imply that the gauge couplings $g_{\rm L}$ and $g_{\rm R}$ of the $SU(2)_{\rm L}$ and $SU(2)_{\rm R}$ subgroups, respectively, are equal. These models are viable when considered in isolation, but run into serious difficulties when they appear as constituents of grand unified theories or when their cosmological implications are treated. For example, the symmetric LRMs exaggerate the value of $\sin^2\theta_{\rm W}$, fail to provide a natural explanation for the cosmological baryon asymmetry, and may lead to problems associated with domain walls in cosmology. These contradictions can be removed by assuming that the discrete LR symmetry is not a good symmetry at low energies, i.e., it is violated at energies whose scale is much larger than the scale at which the $SU(2)_{\rm L} \times SU(2)_{\rm R} \times U(1)_{B-L}$ symmetry is broken. As a result, it turns out that $g_{\rm L} \neq g_{\rm R}$.

The purpose of the present work is to investigate the Higgs sector of the asymmetric LRM. In the next section we shall provide necessary insights into the scalar sector build-up of the model under study and express the Higgs bosons coupling constants through the masses and the mixing angles of the neutrinos. In Sect. 3 we shall consider the detection signatures of the singly charged Higgs bosons in the ultrahigh-energy neutrinos experiments. There we shall also study the production processes of the doubly charged Higgs bosons and investigate the detection signatures of the neutral Higgs bosons in the process of the electron–muon recharge. The analysis of the results obtained is presented in Sect. 4.

2 Description of the model

The subject of our study will be the asymmetric LRM ($g_{\rm L} \neq g_{\rm R}$) [4] with the bidoublet

$$\Phi\left(\frac{1}{2},\frac{1}{2},0\right) = \begin{pmatrix} \Phi_1^0 & \Phi_2^+ \\ \Phi_1^- & \Phi_2^0 \end{pmatrix},$$

[a] E-mail: boyarkin@bspu.unibel.by

and two triplets

$$\Delta_{\rm L}(1,0,2)=\begin{pmatrix}\delta_{\rm L}^{++}\\ \delta_{\rm L}^{+}\\ \delta_{\rm L}^{0}\end{pmatrix},\qquad \Delta_{\rm R}(0,1,2)=\begin{pmatrix}\delta_{\rm R}^{++}\\ \delta_{\rm R}^{+}\\ \delta_{\rm R}^{0}\end{pmatrix}$$

(in brackets the quantum numbers of the $SU(2)_{\rm L}$, $SU(2)_{\rm R}$ and the $U(1)_{B-L}$ groups are given). With this choice of the Higgs sector the neutrino is a Majorana particle.

The spectrum of the physical Higgs bosons consists of: four doubly charged scalars $\Delta_{1,2}^{(\pm\pm)}$, four singly charged scalars $h^{(\pm)}$ and $\tilde{\delta}^{(\pm)}$, four neutral scalars S_i $(i=1,2,3,4)$ and two neutral pseudoscalars $P_{1,2}$. The masses of these bosons are functions of the vacuum expectation values (VEVs) of the neutral components of the Higgs fields,

$$\langle\delta^0_{\rm L,R}\rangle=\frac{v_{\rm L,R}}{\sqrt{2}},\qquad \langle\Phi_1^0\rangle=k_1,\qquad \langle\Phi_2^0\rangle=k_2,$$

where $v_{\rm L} << \max(k_1,k_2) << v_{\rm R}$, and of a large number of self-coupling constants entering the scalar potential.

Let us choose the Higgs potential in the same form as the authours of [5]. Its obvious expression is given in the Appendix. We shall start with a definition of the interaction Lagrangians of the charged Higgs bosons. In the sector of the singly charged scalar bosons we have four physical Higgs bosons:

$$h^{(\pm)}=b\Phi_+^{(\pm)}+\frac{ak_0}{v_{\rm R}}\delta_{\rm R}^{\pm}+\frac{d\beta k_0^2}{(\alpha+\rho_1-\rho_3/2)v_{\rm R}^2}\delta_{\rm L}^{\pm},\qquad(1)$$

$$\tilde{\delta}^{(\pm)}=\frac{a\beta k_0}{(\alpha+\rho_1-\rho_3/2)v_{\rm R}}\delta_{\rm R}^{\pm}-d\delta_{\rm L}^{\pm},\qquad(2)$$

where

$$\Phi_+^{(\pm)}=\frac{k_1\Phi_1^{(\pm)}+k_2\Phi_2^{(\pm)}}{k_+},\quad \Phi_-^{(\pm)}=\frac{k_1\Phi_2^{(\pm)}-k_2\Phi_1^{(\pm)}}{k_+},$$

$$k_\pm=\sqrt{k_1^2\pm k_2^2},\qquad k_0=\frac{k_-^2}{\sqrt{2}k_+},\quad \alpha=\frac{\alpha_3k_+^2}{2k_-^2},$$

$$\beta=\frac{k_+^2(\beta_1k_1+2\beta_3k_2)}{2k_-^2k_0},$$

$$a=\left[1+\left(1+\frac{\beta^2}{(\alpha+\rho_1-\frac{\rho_3}{2})^2}\right)\frac{k_0^2}{v_{\rm R}^2}\right]^{-1/2},$$

$$b=\left(1+\frac{k_0^2}{v_{\rm R}^2}\right)^{-1/2},\quad d=\left[1+\frac{\beta^2k_0^2}{\left(\alpha+\rho_1-\frac{\rho_3}{2}\right)^2v_{\rm R}^2}\right]^{-1/2};$$

α_i,ρ_i and β_i are the constants entering the Higgs potential (see (A.1)). The squared masses of these particles are given by

$$m_h^2=\alpha(v_{\rm R}^2+k_0^2)+\frac{\beta^2k_0^2}{\alpha+\rho_1-\rho_3/2},$$

$$m_{\tilde{\delta}}^2=(\rho_3/2-\rho_1)v_{\rm R}^2-\frac{\beta^2k_0^2}{\alpha+\rho_1-\rho_3/2}.$$

We also have four doubly charged physical Higgs bosons $\Delta_{1,2}^{(\pm\pm)}$, which are orthonormal mixtures of the $\delta_{\rm L}^{(\pm\pm)}$ and the $\delta_{\rm R}^{(\pm\pm)}$ states:

$$\Delta_1^{(\pm\pm)}=c_{\theta_d}\delta_{\rm L}^{\pm\pm}+s_{\theta_d}\delta_{\rm R}^{\pm\pm},$$

$$\Delta_2^{(\pm\pm)}=-s_{\theta_d}\delta_{\rm L}^{\pm\pm}+c_{\theta_d}\delta_{\rm R}^{\pm\pm},$$

where $c_{\theta_d}=\cos\theta_d$, $s_{\theta_d}=\sin\theta_d$ and

$$\tan 2\theta_d=\frac{2k_-^2[\beta_3k_+^2+\beta_1k_1k_2)}{k_1^2(2\rho_1-\rho_3-4\rho_2)v_{\rm R}^2}.$$

Their masses are given by

$$m_{\Delta_1}^2=\frac{\alpha_3k_-^2+4\rho_2v_{\rm R}^2}{2}+\frac{k_-^4(\beta_3k_+^2+\beta_1k_1k_2)^2}{2k_1^4(4\rho_2+\rho_3-2\rho_1)v_{\rm R}^2},$$

$$m_{\Delta_2}^2=\frac{\alpha_3k_-^2-(2\rho_1-\rho_3)v_{\rm R}^2}{2}-\frac{k_-^4(\beta_3k_+^2+\beta_1k_1k_2)^2}{2k_1^4(4\rho_2+\rho_3-2\rho_1)v_{\rm R}^2}.$$

We shall take the Yukawa Lagrangian describing the gauge invariant interaction in the lepton sector in the following form:

$$\mathcal{L}_Y=-\sum_{a,b}\{h_{ab}\overline{\Psi}_{a\rm L}\Phi\Psi_{b\rm R}+h'_{ab}\overline{\Psi}_{a\rm L}\tilde{\Phi}\Psi_{b\rm R}$$
$$+\mathrm{i}f_{ab}[\Psi^T_{a\rm L}C\tau_2(\vec{\tau}\cdot\vec{\Delta}_{\rm L})\Psi_{b\rm L}+(\mathrm{L}\to\mathrm{R})]+\mathrm{conj.}\},\ (3)$$

where $\Psi_{a\rm L}$ ($\Psi_{a\rm R}$) describes the left-handed (right-handed) lepton doublet, $\tau_{1,2,3}$ are the Pauli matrices, C is the charge conjugation matrix, $\tilde{\Phi}=\tau_2\Phi^*\tau_2$, a and b label different generations, and h_{ab},h'_{ab} and $f_{ab}=f_{ba}$ are the bidoublet and the triplet coupling constants determining the interactions between leptons and the Higgs bosons.

Using (1)–(3) we obtain the Lagrangian which describes the interaction between leptons and the singly charged Higgs bosons

$$\mathcal{L}_l^{sc}$$
$$=\sum_{a,b}\left\{b\left[\frac{h'_{ab}k_2-h_{ab}k_1}{2k_+}\overline{\nu}_a(1-\gamma_5)l_b\right.\right.$$
$$\left.-\frac{h_{ab}k_2-h'_{ab}k_1}{2k_+}\overline{N}_a(1+\gamma_5)l_b\right]h^{(+)}$$
$$+\frac{f_{ab}}{\sqrt{2}}\overline{l^c_a}(1+\gamma_5)\nu_b\left[\left(\frac{d\beta k_0^2}{(\alpha+\rho_1-\rho_3/2)v_{\rm R}^2}h^{(+)}-d\tilde{\delta}^{(+)}\right)\right.$$
$$+\overline{l^c_a}(1-\gamma_5)N_b\left(\frac{ak_0}{v_{\rm R}}h^{(+)}\right.$$
$$\left.\left.\left.+\frac{a\beta k_0}{(\alpha+\rho_1-\rho_3/2)v_{\rm R}}\tilde{\delta}^{(+)}\right)\right]+\mathrm{conj.}\right\},\qquad(4)$$

where the superscript c denotes the operation of the charge conjugation.

With the help of the identity

$$\overline{\nu}_a(1-\gamma_5)l_b=\overline{(\nu_a^c)^c}(1-\gamma_5)(l_b^c)^c$$
$$=-(\nu_a^c)^TC^{-1}(1-\gamma_5)C(\overline{l_b^c})^T=\overline{l_b^c}(1-\gamma_5)\nu_a^c,$$

and the Majorana condition

$$\nu_a^c = \lambda_{\nu a}^* \nu_a,$$

where $\lambda_{\nu a}$ is the creation phase factor of the field ν_a, the first two terms in (4) can be rewritten in the form

$$\sum_{a,b} b \left[\lambda_{\nu a}^* \frac{h'_{ab}k_2 - h_{ab}k_1}{k_+} \overline{l_b^c}(1-\gamma_5)\nu_a \right.$$
$$\left. -\lambda_{Na}^* \frac{h_{ab}k_2 - h'_{ab}k_1}{k_+} \overline{l_b^c}(1+\gamma_5)N_a \right] h^{(+)}.$$

At $v_\mathrm{L} = 0$ the interaction of the singly charged Higgs bosons with the W_1 and the Z_1 bosons is determined only by the Lagrangian

$$\mathcal{L}_V^{sc} = \left\{ -\frac{g_\mathrm{L} g_\mathrm{R} c_W c_\Phi b k_-^2}{\sqrt{2}k_+} h^{(+)} \right.$$
$$-\frac{g_\mathrm{L} g_\mathrm{R} g' s_W (g'^{-1} s_W c_\Phi + g_\mathrm{R}^{-1} s_\Phi)}{c_W}$$
$$\left. \times [a k_0 h^{(+)} + \frac{a\beta k_0}{(\alpha + \rho_1 - \frac{\rho_3}{2})}\tilde{\delta}^{(+)}] \right\} W_{\mathrm{R}\mu} Z_{1\mu}$$
$$+\mathrm{conj.}, \qquad (5)$$

where $c_W = \cos\theta_\mathrm{W}$, $s_W = \sin\theta_\mathrm{W}$, $c_\Phi = \cos\Phi$, $s_\Phi = \sin\Phi$, $c_\xi = \cos\xi$, $s_\xi = \sin\xi$, $W_{\mathrm{R}\mu} = -s_\xi W_{1\mu} + c_\xi W_{2\mu}$, $W_{\mathrm{L}\mu} = c_\xi W_{1\mu} + s_\xi W_{2\mu}$, Φ and ξ are the mixing angles of the neutral and the charged gauge bosons, respectively, and g' is the gauge coupling of the $U(1)_{B-L}$ subgroup. We recall that the W_1 – and the Z_1 – gauge bosons are the analogs of the gauge bosons of the SM. After taking into consideration the contributions connected with v_L in (5) once more the term appears, namely

$$-\frac{e g_\mathrm{L} s_\Phi v_\mathrm{L}(g' g_\mathrm{R}^{-1} + g_\mathrm{R} g'^{-1})}{c_W}$$
$$\times \left(\frac{d\beta k_0^2}{(\alpha + \rho_1 - \frac{\rho_3}{2})v_\mathrm{R}^2} h^{(+)} - d\tilde{\delta}^{(+)} \right) W_{\mathrm{L}\mu} Z_{1\mu}. \quad (6)$$

At quasi-degeneracy of the bidoublet vacuum expectation values (QDBVEVs)

$$k_1 = k - \frac{\Delta k}{2}, \qquad k_2 = k + \frac{\Delta k}{2}, \qquad \frac{\Delta k}{k} \ll 1,$$

the minimization conditions

$$\frac{\partial V}{\partial \Phi_1^0} = \frac{\partial V}{\partial \Phi_2^0} = 0 \qquad (7)$$

allow one to express the constants $\mu_{1,2}^2$ in the following form:

$$\mu_1^2 = \frac{1}{2}\{\gamma_1 + 2\gamma_2 - \gamma_3 + (\Delta k)^{-1}(k + 2\Delta k)\gamma_4\}, \qquad (8)$$

$$\mu_2^2 = \frac{1}{4}\{\gamma_1 + 2\gamma_5 + \gamma_3 - (\Delta k)^{-1} k \gamma_4\}, \qquad (9)$$

where

$$\gamma_1 = 2k^2(\lambda_1 + 2\lambda_2 + \lambda_3 + 2\lambda_4),$$
$$\gamma_2 = \alpha_1(v_\mathrm{L}^2 + v_\mathrm{R}^2)/2 + \beta_2 v_\mathrm{L} v_\mathrm{R},$$
$$\gamma_3 = 2k^2(-\lambda_1 + 2\lambda_2 + \lambda_3),$$
$$\gamma_4 = \alpha_3(v_\mathrm{L}^2 + v_\mathrm{R}^2)/2 + (\beta_3 - \beta_2) v_\mathrm{L} v_\mathrm{R},$$
$$\gamma_5 = \alpha_2(v_\mathrm{L}^2 + v_\mathrm{R}^2) + \beta_1 v_\mathrm{L} v_\mathrm{R}/2.$$

Recall that (8) and (9) are taken into consideration while obtaining the obvious expression of the Higgs boson mass matrix. In the case of the degeneracy of the bidoublet vacuum expectation values (DBVEVs) ($k_1 = k_2 = k_g$) we should suggest that

$$\gamma_4 = 0. \qquad (10)$$

Then for the squared mass of the $h^{(\pm)}$ boson the calculations produce the following result:

$$m_h^2 \approx \beta_1 v_\mathrm{L} v_\mathrm{R} + \left(\frac{\rho_3}{2} - \rho_1\right) \frac{v_\mathrm{L}^2 v_\mathrm{R}^2}{k^2}.$$

To estimate v_L one can use the inequality

$$v_\mathrm{L}^2 < \rho_t \Delta\rho_0 g_\mathrm{L}^{-1}(k_1^2 + k_2^2), \qquad (11)$$

which follows from the analysis of the CDF and DO experiments concerning the measurement of the parameter ρ_0 [6]:

$$\rho_0 = 1 + \Delta\rho_0 = \frac{m_{W_\mathrm{L}}^2}{c_W^2 m_{Z_1}^2 (1 + \rho_t)},$$

where

$$\rho_t = \frac{3 G_F m_t^2}{8\sqrt{2}\pi^2}.$$

Putting $((\rho_3/2) - \rho_1) \approx \beta_1 \approx 1$, $v_\mathrm{R} \approx 1\,\mathrm{TeV}$ and using for v_L its upper bound, we obtain

$$m_h \leq 140\,\mathrm{GeV}.$$

Thus, even with the DBVEV the $h^{(\pm)}$ boson can be as heavy as the singly charged Higgs boson in the minimal supersymmetric standard model $H^{(\pm)}$ for which as we know the following equation is true:

$$m_H > m_W.$$

It is necessary to stress that the existence of restrictions similar to (10) entirely depends on the Higgs potential choice. Thus, for example, while making the change in (A.1)

$$\Phi^* \longrightarrow \tilde{\Phi}$$

only with the terms being proportional to α_3, the conditions (7) do not lead to any restrictions on the mass of the $h^{(\pm)}$ boson and m_h turns out to be proportional to v_R.

Since the mixing angle in the sector of the charged gauge bosons is defined as

$$\tan 2\xi = \frac{2k_1 k_2}{v_\mathrm{R}^2 - v_\mathrm{L}^2},$$

$$\tan 2\theta_0 = \frac{4k_1k_2k_-^2[2(2\lambda_2+\lambda_3)k_1k_2+\lambda_4k_+^2]}{k_1k_2[(4\lambda_2+2\lambda_3)(k_-^4-4k_1^2k_2^2)-k_+^2(2\lambda_1k_+^2+8\lambda_4k_1k_2)]-\alpha_2v_{\rm R}^2k_+^4}.$$

it has a maximum in the case of DBVEV. However, even if $v_{\rm R}$ is about 1 TeV, ξ_{max} is of order 1.5×10^{-2}; this is in accordance with the present experimental bounds.

If at the DBVEV (QDBVEV) in the quark sector one uses the traditional expression for the Yukawa Lagrangian

$$\mathcal{L}_Y = -\sum_{a,b}(h_{ab}^{(q)}\overline{Q}_{a\rm L}\Phi Q_{b\rm R}+h_{ab}^{(q)\prime}\overline{Q}_{a\rm L}\tilde{\Phi}Q_{b\rm R}+\text{conj.}), \quad (12)$$

where $Q_{a\rm L}$ ($Q_{a\rm R}$) describes the left-handed (right-handed) quark doublet, then one gets the relation

$$\mathcal{M}_u=\mathcal{M}_d, \quad (\mathcal{M}_u\approx\mathcal{M}_d), \quad (13)$$

where $\mathcal{M}_{u,d}$ are the diagonal mass matrices for the up and down quarks. Then, to avoid (13) one can introduce the additional Higgs triplets $\Delta'_{\rm L}(1,0,2/3)$ and $\Delta'_{\rm R}(0,1,2/3)$ and one supplements the Lagrangian (12) with the term [7]

$$-\sum_{a,b}\{f_{ab}^{(q)}Q_{a\rm L}^TC\tau_2(\vec{\tau}\cdot\vec{\Delta}'_{\rm L})Q_{b\rm L}+(\rm L\to R)+\text{conj}\}.$$

There is also another way – introduce the additional bidoublet $\Phi_u(1/2,1/2,0)$ which interacts with both the up and the down quarks, but which contributes only to the up quark mass [8]. However, in both approaches the undesirable increase in number of the physical Higgs bosons takes place. In the asymmetric version of the LRM it is not at all necessary to complicate the Higgs sector to obtain $\mathcal{M}_u\neq\mathcal{M}_d$. Thus, for example, instead of (12) one can take the Lagrangian similar in its structure to the corresponding Lagrangian of the SM

$$\mathcal{L}_Y^{(q)} = -\sum_{a,b}\Big\{h_{ab}^{(q)}\overline{Q}_{a\rm L}\tau_-\Phi\tau_+Q_{b\rm R} + h_{ab}^{(q)\prime}\overline{Q}_{a\rm L}\tau_+\Phi^*\tau_-Q_{b\rm R}+\text{conj.}\Big\}, \quad (14)$$

where $\tau_\pm=(1/2)(\tau_1\pm{\rm i}\tau_2)$. At such a choice of $\mathcal{L}_Y^{(q)}$ the interaction between the singly charged Higgs bosons and quarks does not take place. We note that the Lagrangians (12) and (14) give the same values of the quark masses.

The Lagrangians describing the interactions of leptons and Z_1 bosons with the $\Delta_{1,2}^{(\pm\pm)}$ scalars are given by the expressions

$$\mathcal{L}_l^{dc} = -\sum_{a,b}\frac{f_{ab}}{2}[\overline{l_a^c}(1+\gamma_5)l_bc_{\theta_d}-\overline{l_a^c}(1-\gamma_5)l_bs_{\theta_d}]\Delta_1^{(++)} +(\Delta_1\to\Delta_2,\theta_d\to\theta_d-\frac{\pi}{2})+\text{conj.}, \quad (15)$$

$$\begin{aligned}\mathcal{L}_Z^{dc} = {}&[(\alpha_{\rm L}c_{\theta_d}^2+\alpha_{\rm R}s_{\theta_d}^2)\Delta_1^{(--)}\partial_\mu\Delta_1^{(++)} \\ &+(\alpha_{\rm L}^2c_{\theta_d}^2+\alpha_{\rm R}^2s_{\theta_d}^2)\Delta_1^{(--)}\Delta_1^{(++)}Z_{1\mu} \\ &+(\Delta_1\to\Delta_2,\theta_d\to\theta_d+\pi/2) \\ &-s_{\theta_d}c_{\theta_d}(\alpha_{\rm L}-\alpha_{\rm R})\Delta_1^{(--)}\partial_\mu\Delta_2^{(++)} \\ &-s_{\theta_d}c_{\theta_d}(\alpha_{\rm L}^2-\alpha_{\rm R}^2)\Delta_1^{(--)}\Delta_2^{(++)}Z_{1\mu}+\text{conj.}]Z_{1\mu},\end{aligned} \quad (16)$$

where

$$g'=\frac{1}{\sqrt{c_W^2e^{-2}-g_{\rm R}^{-2}}},$$
$$\alpha_{\rm L}=e[2\cot 2\theta_{\rm W}c_\Phi-g's_\Phi c_W^{-1}g_{\rm R}^{-1}],$$
$$\alpha_{\rm R}=e[-2c_W^{-1}s_Wc_\Phi+s_\Phi c_W^{-1}g'^{-1}g_{\rm R}^{-1}(g_{\rm R}^2-g'^2)].$$

For the Higgs potential (A.1) the matrix of the transition to the mass eigenstate basis for the neutral scalar Higgs boson is too cumbersome. Since we are going to restrict ourselves only to the investigation of reactions in which the singly charged and the neutral Higgs bosons do not appear simultaneously, we can simplify the expression for the Higgs potential. Let us suppose that the following is true:

$$\alpha_1=-\frac{2\alpha_2k_2}{k_1}, \qquad \alpha_3=-\frac{2\alpha_2k_-^2}{k_1k_2}, \qquad \beta_1=-\frac{2\beta_3k_2}{k_1}. \quad (17)$$

Then at $v_{\rm L}=0$ we have four scalar,

$$S_1=\Phi_-^{0\rm r}c_{\theta_0}+\Phi_+^{0\rm r}s_{\theta_0}, \quad S_2=-\Phi_-^{0\rm r}s_{\theta_0}+\Phi_+^{0\rm r}c_{\theta_0},$$
$$S_3=\delta_{\rm R}^{0\rm r}, \quad S_4=\delta_{\rm L}^{0\rm r}, \quad (18)$$

and two pseudoscalar,

$$P_1=\Phi_+^{0\rm i}, \qquad P_2=\delta_{\rm L}^{0\rm i}, \quad (19)$$

neutral physical Higgs bosons. In the formulae (18) and (19) the superscript r (i) means the real (imaginary) part of the corresponding quantity, $c_{\theta_0}=\cos\theta_0$, $s_{\theta_0}=\sin\theta_0$, and (see formula on top of the page)

The squared masses of these bosons are defined by

$$m_{S_1}^2 = 2\lambda_1k_+^2+8k_1^2k_2^2(2\lambda_2+\lambda_3)/k_+^2+8\lambda_4k_1k_2 + \frac{4k_1k_2k_-^4[2(2\lambda_2+\lambda_3)k_1k_2/k_+^2+\lambda_4]^2}{\alpha_2v_{\rm R}^2k_+^2}, \quad (20)$$

$$m_{S_2}^2 = -\frac{\alpha_2v_{\rm R}^2k_+^2}{k_1k_2} - \frac{4k_1k_2k_-^4[2(2\lambda_2+\lambda_3)k_1k_2/k_+^2+\lambda_4]^2}{\alpha_2v_{\rm R}^2k_+^2}, \quad (21)$$

$$m_{S_3}^2=2\rho_1v_{\rm R}^2, \quad m_{S_4}=(\rho_3/2-\rho_1)v_{\rm R}^2, \quad (22)$$

$$m_{P_1}^2=2k_+^2(\lambda_3-2\lambda_2)-\frac{\alpha_2v_{\rm R}^2k_+^2}{k_1k_2},$$
$$m_{P_2}^2=(\rho_3/2-\rho_1)v_{\rm R}^2. \quad (23)$$

It should be stressed that (20)–(23) still hold at both the DBVEV and the QDBVEV.

The Lagrangians which determine the interaction of the physical neutral Higgs bosons with leptons and gauge bosons are given by the expression

$$\begin{aligned}
\mathcal{L}_l^n
&= -\frac{1}{\sqrt{2}k_+}\sum_a m_a \bar{l}_{a\mathrm{R}} l_{a\mathrm{L}}(S_1 c_{\theta_0} - S_2 s_{\theta_0}) \\
&\quad -\frac{1}{\sqrt{2}k_+}\sum_{a,b} \bar{l}_{a\mathrm{R}} l_{b\mathrm{L}}[(h_{ab}k_1 - h'_{ab}k_2)(S_1 s_{\theta_0} + S_2 c_{\theta_0}) \\
&\quad +\mathrm{i}(h_{ab}k_1 + h'_{ab}k_2)P_1] \\
&\quad -\frac{1}{\sqrt{2}k_+}\sum_{a,b}\{\overline{N}_{a\mathrm{R}}\nu_{b\mathrm{L}}[h_{ab}(k_1 c_{\theta_0} - k_2 s_{\theta_0}) \\
&\quad +h'_{ab}(k_2 c_{\theta_0} + k_1 s_{\theta_0})]S_1 \\
&\quad -[h_{ab}(k_1 s_{\theta_0} + k_2 c_{\theta_0}) + h'_{ab}(k_2 s_{\theta_0} - k_1 c_{\theta_0})]S_2 \\
&\quad -\mathrm{i}(h_{ab}k_2 + h'_{ab}k_1)P_1\} \\
&\quad -\frac{1}{\sqrt{2}}\sum_{a,b} f_{ab}[\overline{\nu}^c_{a\mathrm{L}}\nu_{b\mathrm{L}}(S_4 + \mathrm{i}P_2) + \overline{N}^c_{a\mathrm{R}}N_{b\mathrm{R}}S_3] \\
&\quad +\mathrm{conj.}, \qquad (24)
\end{aligned}$$

$$\begin{aligned}
2\mathcal{L}_W^n
&= k_+[W^*_{1\mu}W_{1\mu}(g^2_\mathrm{L}c^2_\xi + g^2_\mathrm{R}s^2_\xi) + W^*_{2\mu}W_{2\mu}(g^2_\mathrm{L}s^2_\xi + g^2_\mathrm{R}c^2_\xi) \\
&\quad +\frac{1}{2}s_{2\xi}(g^2_\mathrm{L} - g^2_\mathrm{R})(W^*_{1\mu}W_{2\mu} + W^*_{2\mu}W_{1\mu})](S_1 c_{\theta_0} - S_2 s_{\theta_0}) \\
&\quad -\frac{g_\mathrm{L}g_\mathrm{R}}{k_+}\{[c_{2\xi}(W^*_{2\mu}W_{1\mu} + W^*_{1\mu}W_{2\mu}) \\
&\quad +s_{2\xi}(W^*_{2\mu}W_{2\mu} - W^*_{1\mu}W_{1\mu})][(2k_1k_2 c_{\theta_0} + k^2_- s_{\theta_0})S_1 \\
&\quad -(2k_1k_2 s_{\theta_0} - k^2_- c_{\theta_0})S_2] \\
&\quad -\mathrm{i}g_\mathrm{L}g_\mathrm{R}k_+(W^*_{2\mu}W_{1\mu} - W^*_{1\mu}W_{2\mu})P_1 \\
&\quad +g^2_\mathrm{L}v_\mathrm{L}[W^*_{1\mu}W_{1\mu}c^2_\xi + W^*_{2\mu}W_{2\mu}s^2_\xi \\
&\quad +\frac{1}{2}s_{2\xi}(W^*_{1\mu}W_{2\mu} + W^*_{2\mu}W_{1\mu})]S_4 + g^2_\mathrm{R}v_\mathrm{R}[W^*_{1\mu}W_{1\mu}s^2_\xi \\
&\quad +W^*_{2\mu}W_{2\mu}c^2_\xi - \frac{1}{2}s_{2\xi}(W^*_{1\mu}W_{2\mu} + W^*_{2\mu}W_{1\mu})]S_3, \qquad (25)
\end{aligned}$$

It is known that the interactions in the sector of the neutral gauge bosons are defined by the angle φ_R which is connected with the orientation of the $SU(2)_\mathrm{R}$ generator in the group space. Below we choose φ_R to be equal to zero. We recall that in this case our model will coincide with that proposed by Mohapatra [3]. The reader can find all the Lagrangians describing the interactions of the neutral gauge bosons with the fermions and charged gauge bosons in [4].

The constraints on the Yukawa couplings (YCs) could be obtained by investigation of the neutrino oscillations. Nowadays, there exist three indications in favor of the neutrino oscillation scheme. The first indication comes from the observations of the solar neutrinos in various experiments [9] and their disagreement with the predictions of the standard solar model. The solar neutrino data may be explicable in terms of $\nu_e \to \nu_X$ oscillations $(X = \mu, \tau)$ with either a small mixing angle MSW solution [10]

$$\sin^2 2\theta_{eX} \approx (0.4\text{–}1.3) \times 10^{-2},$$

or a large mixing angle MSW solution

$$\sin^2 2\theta_{eX} \approx (0.5\text{–}0.9),$$

or vacuum oscillations

$$\sin^2 2\theta_{eX} \geq 0.67.$$

The second indication in favor of neutrino mixing comes from the observations of the atmospheric neutrinos by several previous experiments [11] and the most recent confirmation of the earlier results by the Super-Kamiokande Collaboration [12]. The atmospheric neutrino data are explicable by $\nu_\mu \to \nu_\tau$ oscillations with

$$\sin^2 2\theta_{\mu\tau} \geq 0.8.$$

A description in terms of $\nu_\mu \to \nu_e$ oscillations alone fits the data less well, and is in any case largely excluded by the CHOOZ experiment [13]. However, there may be some admixture of $\nu_\mu \to \nu_e$ oscillations [14] as well.

The third indication was obtained in the Los Alamos liquid scintillation neutrino detector (LSND) experiment which gave the first laboratory evidence for the oscillation of both $\overline{\nu}_\mu \to \overline{\nu}_e$ as well as $\nu_\mu \to \nu_e$ [15]. The interpretation of the LSND data favors the choice

$$0.002 \leq \sin^2 2\theta_{e\mu} \leq 0.03.$$

The experiment KARMEN 2 [16] (the second accelerator experiment at medium energies) is also sensitive to this region of parameter space and restricts the allowed values to a relatively small subset of the above region.

Most likely the final determination of the oscillation channels could be done in future high statistics short- and long-baseline oscillation experiments in which neutrino beams are produced at high-energy accelerators, and oscillations could be detected at distant undergroud detectors. Such experiments include the KEK–Kamiokande K2K Collaboration [17], the Fermilab–Saudan MINOS Collaboration [18], and the CERN–Gran Sasso ICARUS, Super-ICARUS, AQUA-RICH, NICE, NOE and OPERA Collaborations [19]. Experiments using a muon storage ring [20] should be especially important due their ability to measure both $\nu_e \to \nu_\mu$, and/or ν_τ, and $\nu_\mu \to \nu_e$ and/or ν_τ, as well as the corresponding oscillation channels for antineutrinos.

We shall represent the relations connecting the neutrino oscillation parameters and the YCs in two-flavor approximation. The neutrino mass matrix in the basis

$$\Psi^T = \left(\nu^T_{e\mathrm{L}}, N^T_{e\mathrm{R}}, \nu^T_{XL}, N^T_{XR}\right)$$

has the form

$$\mathcal{M} = \begin{pmatrix} f_{ee}v_\mathrm{L} & m^e_D & f_{eX}v_\mathrm{L} & M_D \\ m^e_D & f_{ee}v_\mathrm{R} & M'_D & f_{eX}v_\mathrm{R} \\ f_{eX}v_\mathrm{L} & M'_D & f_{XX}v_\mathrm{L} & m^X_D \\ M_D & f_{eX}v_\mathrm{R} & m^X_D & f_{XX}v_\mathrm{R} \end{pmatrix},$$

where $m_D^l = h_{ll}k_1 + h'_{ll}k_2$, $M_D = h_{eX}k_1 + h'_{eX}k_2$, $M'_D = h_{Xe}k_1 + h'_{Xe}k_2$, $l = e, X$. Recall that the constants h_{ll} and h'_{ll} define the charged leptons masses according to the relation

$$m_l = h_{ll}k_2 + h'_{ll}k_1.$$

Let us assume a mixing scheme in which the transition to the eigenstate neutrino mass basis m_i $(i = 1, 2, 3, 4)$ is carried out by the matrix [21]

$$U = \begin{pmatrix} c_{\varphi_e}c_{\theta_\nu} & s_{\varphi_e}c_{\theta_N} & c_{\varphi_e}s_{\theta_\nu} & s_{\varphi_e}s_{\theta_N} \\ -s_{\varphi_e}c_{\theta_\nu} & c_{\varphi_e}c_{\theta_N} & -s_{\varphi_e}s_{\theta_\nu} & c_{\varphi_e}s_{\theta_N} \\ -c_{\varphi_X}s_{\theta_\nu} & -s_{\varphi_X}s_{\theta_N} & c_{\varphi_X}c_{\theta_\nu} & s_{\varphi_X}c_{\theta_N} \\ s_{\varphi_X}s_{\theta_\nu} & -c_{\varphi_X}s_{\theta_N} & -s_{\varphi_X}c_{\theta_\nu} & c_{\varphi_X}c_{\theta_N} \end{pmatrix},$$

where φ_l is the mixing angle inside the lth generation, $\theta_\nu(\theta_N)$ is the mixing angle between the light (heavy) neutrinos belonging to the e and X generations, $c_{\varphi_e} = \cos\varphi_e$, $s_{\varphi_e} = \sin\varphi_e$, and so on.

Using the eigenvalue equation for the mass matrix we obtain the relations which define the YCs through the masses and mixing angles of the neutrino,

$$\begin{aligned} m_D^e &= c_{\varphi_e}s_{\varphi_e} \\ &\quad\times(-m_1c^2_{\theta_\nu} - m_3s^2_{\theta_\nu} + m_2c^2_{\theta_N} + m_4s^2_{\theta_N}), &(26)\\ M_D &= c_{\varphi_e}s_{\varphi_X}c_{\theta_\nu}s_{\theta_\nu}(m_1 - m_3) \\ &\quad+s_{\varphi_e}c_{\varphi_X}c_{\theta_N}s_{\theta_N}(m_4 - m_2), &(27)\\ f_{eX}v_R &= s_{\varphi_e}s_{\varphi_X}c_{\theta_\nu}s_{\theta_\nu}(m_3 - m_1) \\ &\quad+c_{\varphi_e}c_{\varphi_X}c_{\theta_N}s_{\theta_N}(m_4 - m_2), &(28)\\ f_{ee}v_R &= (s_{\varphi_e}c_{\theta_\nu})^2m_1 + (c_{\varphi_e}c_{\theta_N})^2m_2 \\ &\quad+(s_{\varphi_e}s_{\theta_\nu})^2m_3 + (c_{\varphi_e}s_{\theta_N})^2m_4, &(29)\\ f_{XX}v_R &= (s_{\varphi_X}s_{\theta_\nu})^2m_1 + (c_{\varphi_X}s_{\theta_N})^2m_2 \\ &\quad+(s_{\varphi_X}c_{\theta_\nu})^2m_3 + (c_{\varphi_X}c_{\theta_N})^2m_4, &(30)\\ m_D^X &= m_D^e\left(\varphi_e \to \varphi_X, \theta_{\nu,N} \to \theta_{\nu,N} + \frac{\pi}{2}\right), \\ M'_D &= M_D(\varphi_e \leftrightarrow \varphi_X). \end{aligned}$$

The change L $\to$ R in the left-hand sides of (28)–(30) results in the change $\varphi_{e,X} \to \varphi_{e,X} + \pi/2$ in their right-hand sides.

It should be stressed that in the LRM even if the mixing angle between the light neutrinos belonging to the e and X generations θ_ν is equal to zero, the non-diagonal YCs could be not equal to zero.

Let us show that assuming the "sea-saw" relation to be obeyed,

$$m_{\nu_l}m_{N_l} = m_l^2,$$

we can have both small and large mixing angles between the light and the heavy neutrinos φ_l. This could easily be done in a one-flavor approximation. Thus, demanding

$$m_D^l = m_l, \qquad v_L \to 0, \tag{31}$$

we have the expression

$$\tan 2\varphi_l \approx 2\left(\frac{m_{\nu_l}}{m_{N_l}}\right)^{1/2}. \tag{32}$$

On the other hand, having suggested that

$$(m_D^l)^2 - f_{ll}^2v_Rv_L = m_l^2, \tag{33}$$

to define the angle φ_l we obtain the formula

$$\tan 2\varphi_l \approx 2\left(\left(\frac{m_l}{f_{ll}v_R}\right)^2 + \frac{v_L}{v_R}\right)^{1/2}. \tag{34}$$

If one assumes the mass of the W_2 boson to be equal to 800 GeV, as we are doing hereafter, then with the help of the inequality

$$m_{N_e} < 63\,\text{GeV}\left(\frac{1.6\,\text{TeV}}{m_{W_2}}\right)^4,$$

resulting from the experiments aimed at finding the neutrino-less double β decay, for the heavy electron neutrino mass we obtain a value of order 1 TeV. It is reasonable to assume that m_{N_X} has the same order of magnitude. To find v_R one can use the relation

$$v_R = \sqrt{\frac{4(m^2_{W_2} - m^2_{W_1})}{g_L^2(2 + \tan^2 2\xi)}},$$

following from the definition of the $W_{1,2}$ bosons masses. In our case we obtain v_R equal to 1.7 TeV.

Equation (32) will always lead to small values of the angles φ_l. However, (34) can produce not only small angles φ_l, but also large ones. Actually, let $\Delta\rho_0$ be equal to 10^{-4}, then v_L can reach the values of order 1.7 GeV, and we find φ_l to be of order 3×10^{-2}. Another possible version with $v_L = 0$ always generates small values of φ_l. Therefore, we should distinguish two possibilities: (i) the mixing angles inside the neutrino generation are small (10^{-5}–10^{-6}), which occurs at both $v_L = 0$ and $v_L \neq 0$; (ii) at non-zero values of v_L, the values of the angles φ_l can be as large as a few $\times 10^{-2}$.

The best laboratory limits on the left-handed neutrino masses can be summarized as follows [6]:

$$m_{\nu_e} \le 5\,\text{eV}, \qquad m_{\nu_\mu} \le 170\,\text{keV}, \qquad m_{\nu_\tau} \le 18.2\,\text{MeV}.$$

These are most independent of the model as they follow purely from the kinematics. The mass of the electron neutrino (more precisely the electron antineutrino) has been investigated using the β^- decay of tritium:

$$^3_1\text{H} \to {}^3_2\text{He} + e^- + \overline{\nu}_e.$$

The limits for the mass of the ν_μ comes from the study of the decay

$$\pi^+ \to \mu^+ + \nu_\mu,$$

with the pion at rest. The bound on the ν_τ mass follows from the investigation of the decays

$$\tau^- \to KK\nu_\tau, \quad 5\pi\nu_\tau, \quad 6\pi\nu_\tau.$$

To make an estimate of the YCs below, we shall use for the left-handed neutrino masses and the interfamily mixing angle θ_ν their upper experimental bounds ($\theta_\nu \leq 0.03$) and assume

$$\theta_\nu = \theta_N, \qquad m_{N_e} = 1\,\mathrm{TeV}, \qquad m_{N_X} = 1.5\,\mathrm{TeV}.$$

To suppress the oscillations between the charged leptons it is necessary that the bidoublet non-diagonal constants h_{eX} and h'_{eX} satisfy the relation

$$k_2 h_{eX} + k_1 h'_{eX} = 0.$$

Then, taking into consideration the definitions of m_{W_1} and M_D, we can express h'_{eX}, h'_{Xe}, h_{eX} and h_{Xe} as a function of k_1:

$$h'_{eX} = \Omega, \quad h_{eX} = -\frac{\Omega k_1}{k_2},$$
$$h'_{Xe}(h_{Xe}) = h'_{eX}(h_{eX})\{M_D \longrightarrow M'_D\}, \qquad (35)$$

where

$$\Omega = \frac{\sqrt{(1+\rho_t\Delta\rho_0 - 4k_1^2/v_\mathrm{R}^2)[2g_\mathrm{L}^{-2}m_{W_1}^2 - k_1^2(1+\rho_t\Delta\rho_0)]}M_D}{2g_\mathrm{L}^{-2}m_{W_1}^2 - 2k_1^2(1+\rho_t\Delta\rho_0) + 4k_1^4/v_\mathrm{R}^2},$$

and the values of k_1 are within the range from 0 to $2g_\mathrm{L}^{-2}m_{W_1}^2/(1+\rho_t\Delta\rho_0)$. In (35) we have changed v_L by the upper bound which follows from (11).

From the expression for the Yukawa Lagrangian (3) follows that in the lepton sector the LRM will generate both the flavor changing neutral currents (FCNCs) and the flavor changing charged currents (FCCCs) at the tree level. The Higgs doublet of the SM does not generate a tree level FCNC because the mass matrix is directly proportional to the Yukawa coupling matrix, so diagonalization of the former automatically diagonalizes the latter. However, in the LRM, the mass matrix is the sum of the four Yukawa coupling matrices (each times the appropriate VEV), and since the Yukawa coupling matrices are generally not simultaneously diagonalizable, diagonalization of the mass matrix will not, as a rule, diagonalize the Yukawa coupling matrices, leading both to the FCNC and to the FCCC at the tree level. The LRM structure allows one to obtain sizable values of the bidoublet non-diagonal constants not only at large values of φ_e and φ_X, but at both $M_D = 0$ (DBVEV) and $M_D \approx 0$ (QDBVEV). Thus, for example, at the DBVEV these constants become arbitrary because the equality of their sum to zero is the only demand they must satisfy. On the other hand, investigating the cases of the large mixing angles, the DBVEV and the QDBVEV we must consider only such values of the lepton non-diagonal couplings h_{eX}, h'_{eX} and the physical Higgs bosons masses which do not conflict with the experimentally obtained bounds both on the FCNC and on the FCCC. Up to now there are several experimental tests of lepton number violating interactions mediated by the virtual doubly charged Higgs bosons of the LRM (see [22] and references therein) which lead to constraints on the triplet YCs and the doubly charged Higgs bosons masses. Let us investigate two lepton number violating processes which could give us information about the bidoublet YCs and the mass values both of the neutral and of the singly charged Higgs bosons. First, we consider the decay

$$\mu^- \to e^-\nu_e\overline{\nu}_\mu. \qquad (36)$$

For the case of the right polarized μ^- we get

$$\Gamma_{\mu^-\to e^-\nu_e\overline{\nu}_\mu} = \frac{d^4(f_{ee}f_{\mu\mu})^2}{96(2\pi)^3 m_{\tilde{\delta}}^4} m_\mu^5. \qquad (37)$$

Using the current data [6]

$$\frac{\Gamma_{\mu^-\to e^-\nu_e\overline{\nu}_\mu}}{\Gamma_{\mu^-\to all}} < 1.2\times 10^{-2},$$

we are lead to the following bounds on the FCCC parameters:

$$d^2 f_{ee}f_{\mu\mu} < 0.746\times 10^{-3}\left(\frac{m_{\tilde{\delta}}}{\mathrm{GeV}}\right)^2. \qquad (38)$$

Having done the analogous calculations for the decay of the left polarized muon, we obtain

$$b^2\alpha_{ee}\alpha_{\mu\mu} < 0.373\times 10^{-3}\left(\frac{m_h}{\mathrm{GeV}}\right)^2, \qquad (39)$$

where

$$\alpha_{ab} = \frac{h'_{ab}k_2 - h_{ab}k_1}{k_+}.$$

The most stringent constraint could be found from the upper limit for the lepton flavor changing decay

$$\mu^- \to e^+e^-e^-. \qquad (40)$$

The detailed calculations of its width can be done taking into account the Feynman diagrams with the $\Delta_1^{(--)}$-, S_1- and S_2- exchanges, which yield

$$\Gamma_{\mu^-\to e^+e^-e^-} = \frac{\tau m_\mu^5}{96(2\pi)^3}, \qquad (41)$$

where

$$\tau = \left(\frac{f_{ee}f_{e\mu}}{m_{\Delta_1}^2}\right)^2 + \frac{1}{8k_+^2}\left[\alpha_{e\mu}^2\left(\frac{m_e c_{\theta_0}s_{\theta_0} - \alpha_{ee}k_+ s_{\theta_0}^2}{m_{S_1}^2} - \frac{m_e s_{\theta_0}c_{\theta_0} + \alpha_{ee}k_+ c_{\theta_0}^2}{m_{S_2}^2}\right)^2 + \frac{(h_{ee}k_1 + h'_{ee}k_2)^2(h_{e\mu}k_1 + h'_{e\mu}k_2)^2}{k_+^2 m_{P_1}^4}\right]$$
$$+\frac{2m_e f_{ee}f_{e\mu}}{m_\mu m_{\Delta_1}^2 k_+}\left\{\alpha_{e\mu}\left[\left(\frac{m_e c_{\theta_0}s_{\theta_0} - \alpha_{ee}k_+ s_{\theta_0}^2}{m_{S_1}^2} - \frac{m_e s_{\theta_0}c_{\theta_0} + \alpha_{ee}k_+ c_{\theta_0}^2}{m_{S_2}^2}\right)\right] - \frac{(h_{ee}k_1 + h'_{ee}k_2)(h_{e\mu}k_1 + h'_{e\mu}k_2)}{k_+ m_{P_1}^2}\right\},$$

and for the sake of simplicity we set $\theta_d = 0$. Experimental constraints [6] thus imply that the LRM parameters must satisfy

$$\sqrt{\tau} < 2.32 \times 10^{-11}\,\mathrm{GeV}^{-2}. \qquad (42)$$

Now we could estimate the values of the lepton bidoublet YCs at the DBVEV. Since in this case $\theta_0 = 0$, and $h_{e\mu} = -h'_{e\mu}$, the contributions connected with the S_1 and P_1 Higgs bosons have disappeared. Having assumed that $f_{e\mu}$ is equal to zero and m_{S_2} is of the order of 1 TeV, we obtain

$$h_{e\mu} h_{ee} < 3.25 \times 10^{-5}. \qquad (43)$$

The bounds on the neutral Higgs bosons masses may be found from the investigation of the $\overline{K}^0 \leftrightarrow K^0$ transitions as well. We recall that for the model under study the two kind of Yukawa Lagrangians $\mathcal{L}_Y^{(q)}$ describing the gauge invariant interaction in the quark sector could be used. Thus, we have

$$\begin{aligned}\mathcal{L}_q^n = &-\frac{1}{\sqrt{2}k_+}\sum_{a,b}\overline{u}_a\Bigg\{\Bigg[m_{u_a}(c_{\theta_0} - \frac{2k_1k_2}{k_-^2}s_{\theta_0})S_1 \\ &-m_{u_a}(s_{\theta_0} + \frac{2k_1k_2}{k_-^2}c_{\theta_0})S_2 - \mathrm{i}m_{d_a}\gamma_5 P_1\Bigg]\delta_{ab} \\ &+\frac{k_+^2}{k_-^2}\left(\mathcal{K}\mathcal{M}_d\mathcal{K}^*\right)_{ab}\left(S_1 s_{\theta_0} + S_2 c_{\theta_0}\right)\Bigg\} u_b \\ &+(u_a \to d_a, m_{u_a} \leftrightarrow m_{d_a}, \gamma_5 \to -\gamma_5), \end{aligned} \qquad (44)$$

for the case when $\mathcal{L}_Y^{(q)}$ is given by (12), and

$$\begin{aligned}\mathcal{L}_q^n = &-\frac{1}{\sqrt{2}k_+}\sum_{a}\overline{u}_a\Bigg\{m_{u_a}\Bigg[\left(c_{\theta_0} + \frac{k_1}{k_2}s_{\theta_0}\right)S_1 \\ &-\left(s_{\theta_0} - \frac{k_1}{k_2}c_{\theta_0}\right)S_2\Bigg] + \frac{\mathrm{i}m_{u_a}k_1}{k_2}\gamma_5 P_1\Bigg\} u_a \\ &+(u_a \to d_a, \theta_0 \to -\theta_0), \end{aligned} \qquad (45)$$

when (14) is used for the definition of $\mathcal{L}_Y^{(q)}$. From (44) and (45) follows that for our choice of the Yukawa potential (see (17)) only the Lagrangian (44) contains the flavor violating couplings which could generate large $\overline{K}^0 \leftrightarrow K^0$ transitions in contradiction to experiment. Therefore, the bounds on the masses of the Higgs bosons giving rise to the FCNC in the quark sector could be obtained for the case when $\mathcal{L}_Y^{(q)}$ is determined exclusively by (12). Due to the tiny angle θ_0 ($\theta_0 < k_+^2/v_\mathrm{R}^2$), the main contribution to the $\overline{K}^0$–K^0 mixing comes from the diagram with the S_2 exchange. It can be shown that in this case the bound on the S_2 boson mass will be given by

$$m_{S_2} \geq 10\,\mathrm{TeV}. \qquad (46)$$

3 Higgs phenomenology

At present the most powerful neutrino accelerators are the cosmic accelerators in the outer space. Some are known sources of ultrahigh-energy (UHE) cosmic ray neutrinos. One of them exploits neutrinos produced when UHE protons inelastically scatter off the cosmic microwave background radiation (CBR) in processes such as $\gamma p \to n\pi^+$ where the produced pion subsequently decays. The CBR neutrino energy can be as large as 1 EeV. Other sources of UHE neutrinos are the active galactic nuclei (AGN). Typical AGN luminosities are in the range 10^{44} to 10^{47} erg/s.

From this tremendous power output one infers that the source powering AGN is gravity, i.e. matter accretion into a supermassive ($M \geq 10^6 M_\odot$) black hole (such objects are known to exist in the centers of nearby galaxies observed with the Hubble space telescope). Within AGN, protons accelerated to a very high energy interact with matter or ambient radiation and produce pions whose decay products include photons and neutrinos. The maximum energy of AGN neutrinos is of the order of 1 PeV. There are a lot of papers where the diffuse fluxes both of the CBR and the AGN neutrinos have been estimated (see [23], and references therein). Let us discuss two processes with UHE cosmic neutrinos which could be studied with the help of such neutrino telescopes as DUMAND, BAIKAL NT-200, NESTOR and AMANDA.

The first one is

$$e^-\nu_e \to W_1^- Z_1, \qquad (47)$$

In Fig. 1 the Feynman diagrams corresponding to the process (47) in the second order of perturbation theory are shown. For reasons of convenience we shall investigate this process for the left and the right polarized target electrons separately. First, we focus our attention on the former case. Let us assume that we deal with a right polarized neutrino beam. Then this process is described by the diagrams pictured in Figs. 1a,b,c. Since we have been neglecting the neutrino mass, the cross section will have the same form as that of the reaction

$$e^-\overline{\nu}_e \to W_1^- Z_1 \qquad (48)$$

when the neutrino is of a Dirac nature. For the SM the reaction (48) was considered in [24] while for the version of the LRM suggested by Mohapatra and Sidhu [25] it was investigated in [26]. Since for the right polarized neutrino beam there are no diagrams with virtual Higgs bosons and there are no resonance peaks we shall not discuss this case. We only note that the total cross section is of order 8–10 pb.

In the case of the left polarized neutrino beam the contributions to the cross section of the reaction (47) come from the diagrams shown in Figs. 1c,d. The total cross section is given by the expression

$$\begin{aligned}\sigma_{\mathrm{LL}} \\ = \frac{g_\mathrm{L}^2\beta_{WZ}}{128\pi s}&\Bigg(g_{A1}^2 c_\xi^2 \sum_{i,j=1}^{4}(\varLambda_{ee}^{LL})_i m_i (\varLambda_{ee}^{LL})_j m_j \mathcal{M}_1(m_i, m_j) \\ &+\frac{f_{ee}s_W}{c_W\left[(s-m_{\tilde{\delta}}^2)^2 + \varGamma_{\tilde{\delta}}^2 m_{\tilde{\delta}}^2\right]}\end{aligned}$$

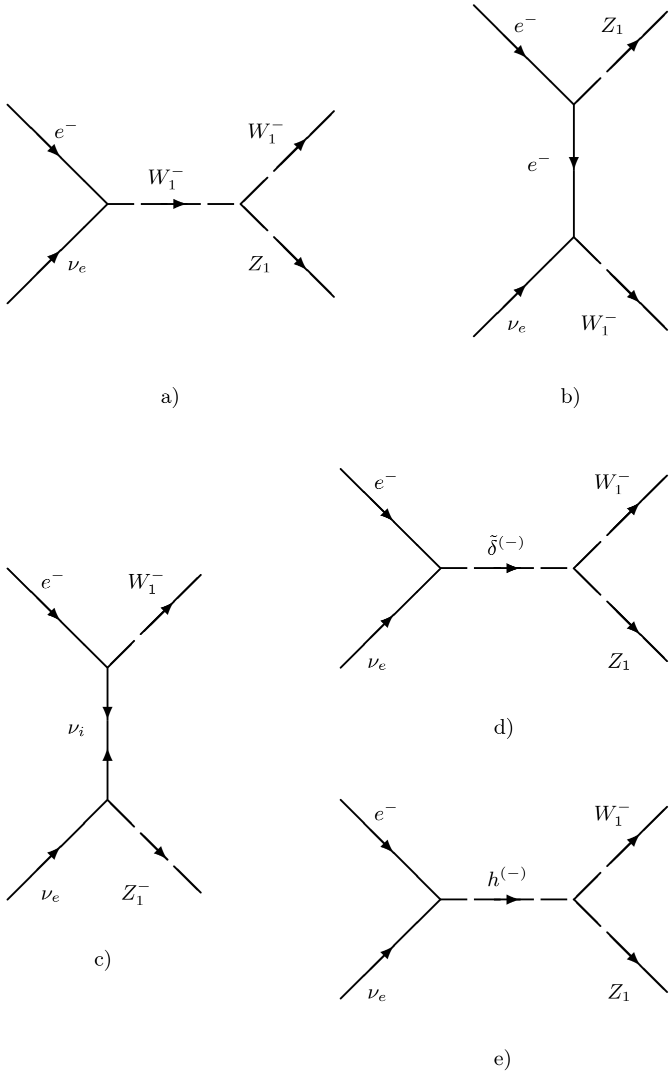

Fig. 1. The Feynman diagrams giving the contributions to the reaction $e^-\nu_e \to W_1^- Z_1$

$$
\begin{aligned}
&\times\left(\frac{g_\mathrm{R} s_W c_\Phi s_\xi d\alpha\beta k_0}{\alpha+\rho_1-\rho_3/2}+\left(\frac{g'}{g_\mathrm{R}}+\frac{g_\mathrm{R}}{g'}\right) g_\mathrm{L} s_\Phi c_\xi d^2 v_\mathrm{L}\right)\\
&\times\Bigg(g_{A1} c_\xi \sum_i (\Lambda_{ee}^{LL})_i m_i (s-m_{\tilde\delta}^2)\mathcal{M}_2(m_i)\\
&+\frac{f_{ee} s_W}{c_W}\left(\frac{g_\mathrm{R} s_W c_\Phi s_\xi d\alpha\beta k_0}{\alpha+\rho_1-\rho_3/2}\right.\\
&\left.+\left(\frac{g'}{g_\mathrm{R}}+\frac{g_\mathrm{R}}{g'}\right) g_\mathrm{L} s_\Phi c_\xi d^2 v_\mathrm{L}\right)\mathcal{M}_3\Bigg)\Bigg), \qquad (49)
\end{aligned}
$$

where

$$
\begin{aligned}
\beta_{WZ} &= \sqrt{\left(1-\frac{m_{W_1}^2+m_{Z_1}^2}{s}\right)^2-\left(\frac{2m_{W_1}m_{Z_1}}{s}\right)^2},\\
\mathcal{M}_1(m_i,m_j) &= \frac{8(L_i-L_j)}{s\delta m_{ij}^2\beta_{WZ}}\left[s+\frac{s(m_i^4+m_j^4)}{8m_{W_1}^2 m_{Z_1}^2}\right.\\
&\left.+\frac{(m_{W_1}^2+m_{Z_1}^2)C_{ij}}{2m_{W_1}^2 m_{Z_1}^2}\right]\\
&+\frac{L_i+L_j}{s m_{W_1}^2 m_{Z_1}^2\beta_{WZ}}\\
&\times[2(m_{W_1}^2+m_{Z_1}^2)(m_{W_1}^2+m_{Z_1}^2\\
&-s-m_i^2-m_j^2)+s(m_i^2+m_j^2)]\\
&+\frac{2(s-2m_{W_1}^2-2m_{Z_1}^2)}{m_{W_1}^2 m_{Z_1}^2},\\
\mathcal{M}_2(m_i) &= \frac{32L_i}{\beta_{WZ}}\left[1+\frac{(m_{W_1}^2+m_{Z_1}^2-s)m_i^2}{4m_{W_1}^2 m_{Z_1}^2}\right]\\
&+\frac{8s(m_{W_1}^2+m_{Z_1}^2-s)}{m_{W_1}^2 m_{Z_1}^2},\\
\mathcal{M}_3 &= 64s+\frac{8s(m_{W_1}^2+m_{Z_1}^2-s)^2}{m_{W_1}^2 m_{Z_1}^2},\\
C_{ij} &= \frac{1}{2}[(m_{W_1}^2+m_{Z_1}^2-s)(m_i^2+m_j^2)\\
&-m_i^4-m_j^4]-m_{W_1}^2 m_{Z_1}^2,\\
L_i &= \ln\left|\frac{m_{W_1}^2+m_{Z_1}^2-s-2m_i^2+s\beta_{WZ}}{m_{W_1}^2+m_{Z_1}^2-s-2m_i^2-s\beta_{WZ}}\right|,\\
L &= L_i|_{m_i=0}, \quad \delta m_{ij}^2 = m_i^2-m_j^2,\\
g_{A1} &= \frac{e}{2c_W}\left(\frac{c_\Phi}{c_W}+\frac{s_\Phi}{\sqrt{c_W^2 g_\mathrm{R}^2 e^{-2}-1}}\right),
\end{aligned}
$$

$$\sum_i (\Lambda_{ee}^{LL})_i m_i = f_{ee} v_\mathrm{L}$$

and the expression for the total decay width of the $\tilde\delta^{(-)}$ boson by the assumption

$$m_{\tilde\delta} < m_{W_2}+m_{Z_1},\ m_{Z_2}+m_{W_1}$$

is given in the Appendix. In Fig. 2 we present σ_LL as a function of the energy in the center of mass system of the leptons $s^{1/2}$ for different values of k_1 and $m_{\tilde\delta}$ at $c_Y = (d\alpha\beta)/(\alpha+\rho_1-\rho_3/2) = 2$ and $f_{\tau\tau} = 0.9$ in the case of the small angles φ_l. The value of f_{ee} corresponding to our choice of parameters is 0.58. As follows from Fig. 2 the cross section increases moving away k_1 from $k_g \simeq 122.9537\,\mathrm{GeV}$. The analysis also shows that it does not practically depend on the values of both v_L and φ_l.

Note that $\Gamma_{\tilde\delta^{(-)}\to W_1^- Z_1}$ is of order of a few $\times 10^{-2}\,\mathrm{MeV}$ and grows very slowly with the increase of $m_{\tilde\delta}$.

So, the value of $f_{\tau\tau}$ is getting very important when the $\tilde\delta^{(-)}$ decay channels with such heavy particles as W_2, Z_2, N_e, N_μ etc. are closed. Actually, let us consider the case of the DBVEV. Then, the total cross section in the non-resonance region is very small. Notwithstanding this fact, the $\tilde\delta^{(-)}$ resonance could be detected. For example, at $c_Y = 3$, $f_{ee} = 0.2$, $v_\mathrm{L} = 1.7\,\mathrm{GeV}$ and $m_{\tilde\delta} = 200\,\mathrm{GeV}$, the resonance peak height $(\sigma_\mathrm{LL})_{\tilde\delta}$ is 3.6, 24, 15×10^3 fb for $f_{\tau\tau}$ equal to 0.8, 0.5, 0.1, respectively. Special attention must be given to the Higgs potential parameters $\alpha+\rho_1-\rho_3/2$ since they have potent effects on $(\sigma_\mathrm{LL})_{\tilde\delta}$. There are

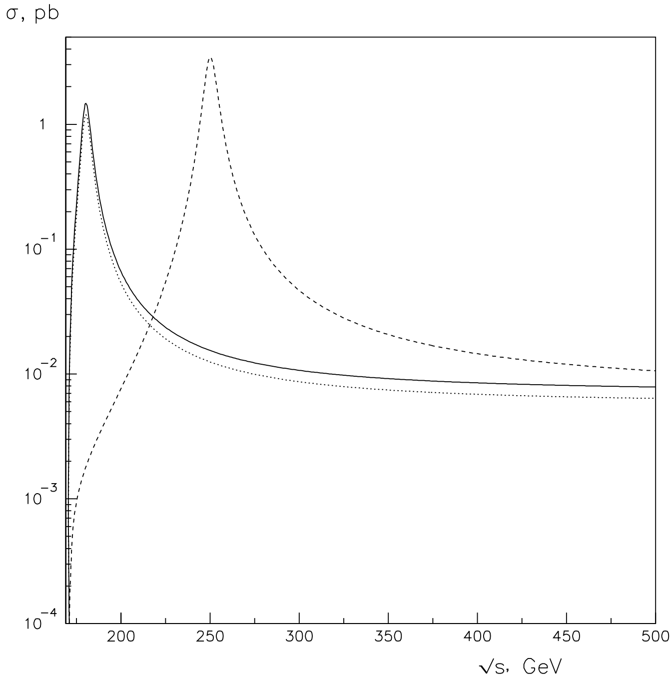

Fig. 2. The total cross section of the reaction $e_L^-\nu_{eL} \to W_1^- Z_1$ as a function of $\sqrt{s}$ at $\Phi = 9.6 \times 10^{-3}$, $\xi = 10^{-2}$, $g_R = 1.4 g_L$, $c_Y = 2$, $\varphi_e = 10^{-6}$, $\varphi_\mu = 2.5 \times 10^{-4}$ and $v_L = 1.7 \times 10^{-7}$ GeV. The solid (dotted) line corresponds to the case $m_{\tilde{\delta}} = 180$ GeV and $k_1 = 10$ GeV ($k_1 = 40$ GeV). The dashed line goes to the case $m_{\tilde{\delta}} = 250$ GeV and $k_1 = 10$ GeV

no reasons that would forbid this combination to be very small. Recall that the sole constraint on the parameters follows from arguments relating to vacuum stability which demand [5]

$$\rho_1 - \rho_3/2 < 0.$$

Then, assuming $\alpha + \rho_1 - \rho_3/2 = 10^{-2}$ and setting the remaining parameters to values which are the same as in the case of Fig. 2 (solid line), we get

$$(\sigma_{\rm LL})_{\tilde{\delta}} = 3.6 \times 10^3 \,\text{pb}.$$

Now we go on to the case when the target electrons are right polarized. The cross section does not equal zero for the right polarized neutrino only. The contributions to it come from the diagrams shown in Figs. 1c,e. It is pertinent to note that in the two Higgs doublet model we also have these diagrams.

The total cross section is defined by the expression

$$\begin{aligned}\sigma_{\rm RR} = \frac{\beta_{WZ}}{256\pi s}\Bigg\{&[m_e g_{\rm L} c_\xi (g_{V1}^e - g_{A1}^e)]^2 \mathcal{M}_1(m_e, m_e) \\ &+ \frac{\kappa}{[(s-m_h^2)^2 + \varGamma_{h^{(-)}}^2 m_h^2]} \\ &\times \left[\frac{m_e g_{\rm L} c_\xi (g_{V1}^e - g_{A1}^e)(s-m_h^2)}{2}\mathcal{M}_2(m_e) + \frac{\kappa}{4}\mathcal{M}_3\right]\Bigg\},\end{aligned} \quad (50)$$

where

$$\begin{aligned} g_{V1}^e &= \frac{e c_\Phi}{c_W s_W}\left(2 s_W^2 - \frac{1}{2}\right) \\ &+ \frac{e s_\Phi}{c_W \sqrt{c_W^2 g_{\rm R}^2 e^{-2} - 1}}\left(\frac{3}{2} - \frac{c_W^2 g_{\rm R}^2}{2e^2}\right), \\ g_{A1}^e &= \frac{e s_\Phi \sqrt{c_W^2 g_{\rm R}^2 e^{-2} - 1}}{2 c_W} - \frac{e c_\Phi}{2 s_W c_W}, \\ \kappa &= \frac{g_{\rm L} g_{\rm R} c_\Phi s_\xi k_-^2 \alpha_{ee}}{c_W k_+}. \end{aligned}$$

Using the definitions of m_l and m_D^l one could find

$$\alpha_{ll} = \frac{2k_1k_2m_l - k_+^2 m_D^l}{k_-^2 k_+}. \quad (51)$$

Unlike the $\tilde{\delta}^{(\pm)}$ boson, the $h^{(\pm)}$ boson could interact with the quarks. When in the quark sector one uses the Yukawa Lagrangian in the form (14), then its interaction with the quarks is absent and there is no restriction on its mass based on a measurement of the inclusive $b \to s\gamma$ cross section. In this case the only restriction will follow from the LEP experiments excluding at 95% CL any charged Higgs with a mass lower than 44.1 GeV [27]. If we use the Yukawa Lagrangian (14) and assume that m_h is less than m_{W_2}, m_{Z_2} and m_{N_l}, the total width of the $h^{(-)}$ Higgs boson is given by

$$\varGamma_{h^{(-)}} = \sum_l \varGamma_{h^{(-)}\to l\nu_{l\rm R}} + \varGamma_{h^{(-)}\to W_1 Z_1};$$

the partial widths are determined in the Appendix. However, for the Yukawa Lagrangian defined by (12) the partial quark widths $\varGamma_{h^{(-)}\to\bar{c}s}$ and $\varGamma_{h^{(-)}\to\bar{t}b}$, where

$$\begin{aligned}\varGamma_{h^{(-)}\to\bar{c}s} &= \frac{3k_+^2}{16\pi m_h \cos^2 2\beta}\left(1 - \frac{(m_c+m_s)^2}{m_h^2}\right) \\ &\times \left((m_c^2 + m_s^2)(1+\sin^2 2\beta) - 4m_c m_s \sin 2\beta\right) \\ &\times w(m_h^2, m_c^2, m_s^2), \\ \varGamma_{h^{(-)}\to\bar{t}b} &= \varGamma_{h^{(-)}\to\bar{c}s}\left(m_c \to m_t, m_s \to m_b\right), \\ \tan\beta &= \frac{k_1}{k_2}\end{aligned}$$

should be taken into account as well.

The analysis shows that the sole chance for the observation of $\sigma_{\rm RR}$ is to operate in the $h^{(-)}$ resonance region. However, the resonance peak height $(\sigma_{\rm RR})_h$ mainly depends on whether the interaction between the $h^{(\pm)}$ Higgs boson and quarks takes place or not. Let us assume that we work with the Yukawa Lagrangian defined by (14). Then the situation is as follows.

In the case of the DBVEV (QDBVEV) and in the case of large values of the angle φ_e we always can detect the resonance peak connected with the $h^{(-)}$ boson. For example, when $m_h = 300$ GeV, $\varphi_e = 2 \times 10^{-2}$ and $k_1 = 140$ GeV, $(\sigma_{\rm RR})_h$ is of order 3.5×10^3 pb. When the angle φ_e is very small then our possibilities for detecting the $h^{(-)}$ resonance peak crucially depend on the values k_1 ($(\sigma_{\rm RR})_h$ increases or decreases when k_1 approaches to or moves off k_g). For example, at $m_h = 250$ GeV and $\varphi_e = 10^{-5}$ we have $(\sigma_{\rm RR})_h = 2 \times 10^{-4}, 6 \times 10^{-2}, 1.6, 14$ pb for k_1 equal to 10, 70, 100, 110 GeV, respectively.

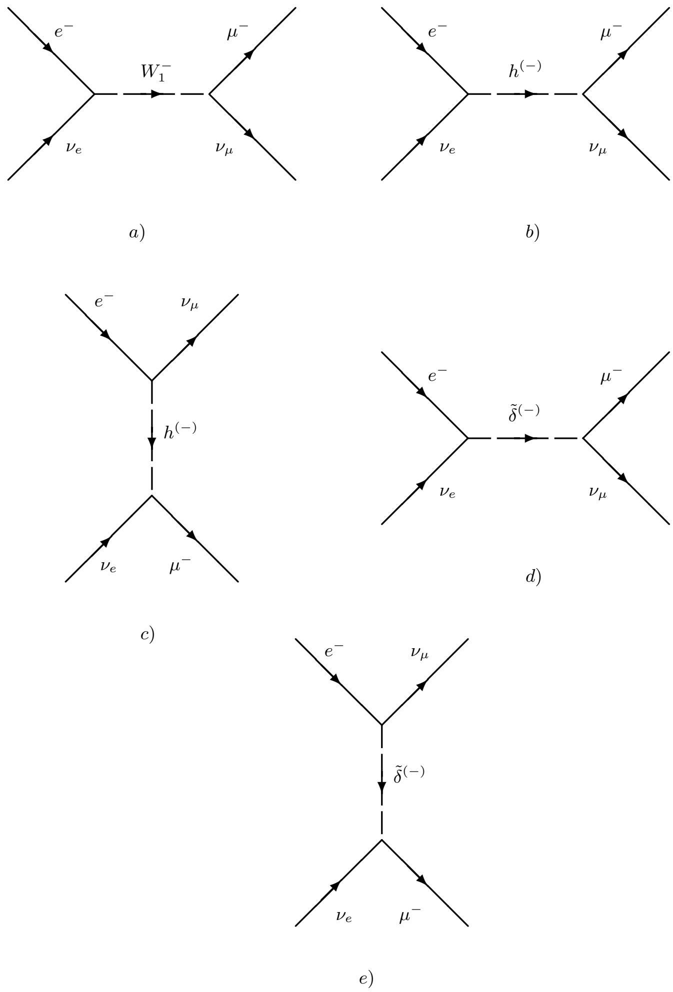

Fig. 3. The Feynman diagrams corresponding to the process $e^-\nu_e \to \mu^-\nu_\mu$

If the gauge invariant interaction between quarks and Higgs bosons are described by (12) then we have no chance to observe the $h^{(-)}$ resonance at $m_h > m_t$. At $m_h < m_t$ $(\sigma_{\rm RR})_h$ reaches its maximum ($\sim few \times 10\,\text{fb}$) for large values of φ_e ($\sim 10^{-2}$) and values of k_1 which are well away from k_g.

Now we consider one more process with the UHE neutrinos, namely

$$e^-\nu_e \to \mu^-\nu_\mu. \qquad (52)$$

The corresponding Feynman diagrams are shown in Fig. 3.

The total cross sections are given by

$$\sigma_{\rm L} = \frac{1}{32\pi s^2} \times \Bigg((1-\lambda_\nu) \Bigg(\frac{g_{\rm L}^4 c_\xi^4 s^3}{3[(s-m_{W_1}^2)^2 + \Gamma_{W_1}^2 m_{W_1}^2]} + \frac{2 g_{\rm L}^2 c_\xi^2 f_{e\mu}^2 (s-m_{W_1}^2)}{[(s-m_{W_1}^2)^2 + \Gamma_{W_1}^2 m_{W_1}^2]} \times \left(\frac{s^2}{2} - m_{\tilde{\delta}}^2 s + \frac{m_{\tilde{\delta}}^2 \Gamma_{\tilde{\delta}^{(-)}}^2 - m_{\tilde{\delta}}^4}{2} N_{\tilde{\delta}} + 2 m_{\tilde{\delta}}^3 \Gamma_{\tilde{\delta}^{(-)}} Q_{\tilde{\delta}} \right)$$

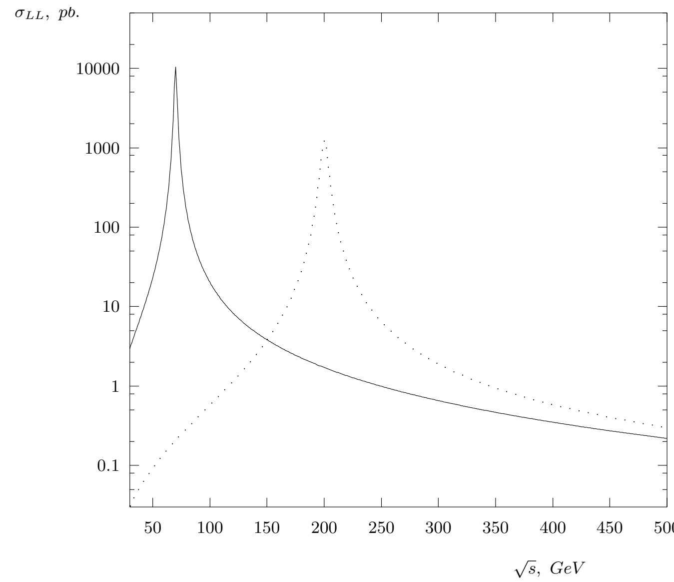

Fig. 4. The σ_{LL} as a function of $\sqrt{s}$ when $\lambda_\nu = 1$, $f_{\tau\tau} = 0.5$, $f_{\mu\mu} = 7.5 \times 10^{-2}$, $f_{ee} = 5.5 \times 10^{-2}$ and $k_1 = 90$ Gev for the cases **a** $m_{\tilde{\delta}} = 200\,\text{GeV}$ (solid line); **b** $m_{\tilde{\delta}} = 70\,\text{GeV}$ (dotted line)

$$+ (f_{e\mu})^2 \left(4 (f_{e\mu})^2 - \frac{2 g_{\rm L}^2 c_\xi^2 \Gamma_{W_1} \Gamma_{\tilde{\delta}^{(-)}} m_{W_1} m_{\tilde{\delta}}}{(s-m_{W_1}^2)^2 + \Gamma_{W_1}^2 m_{W_1}^2} \right) \times \left(s + m_{\tilde{\delta}}^2 N_{\tilde{\delta}} + \frac{m_{\tilde{\delta}}^3 - m_{\tilde{\delta}} \Gamma_{\tilde{\delta}^{(-)}}^2}{\Gamma_{\tilde{\delta}^{(-)}}} Q_{\tilde{\delta}} \right) \Bigg) + \frac{(1+\lambda_\nu) 4 f_{ee}^2 f_{\mu\mu}^2 s^3}{(s-m_{\tilde{\delta}}^2)^2 + \Gamma_{\tilde{\delta}^{(-)}}^2 m_{\tilde{\delta}}^2} \Bigg) \qquad (53)$$

for the case of left polarized electrons, and

$$\sigma_{\rm R} = \frac{1}{128\pi s^2} \left(\frac{(1-\lambda_\nu)(\alpha_{ee}\alpha_{\mu\mu})^2 s^3}{(s-m_h^2)^2 + \Gamma_{h^{(-)}}^2 m_h^2} + (1+\lambda_\nu) \alpha_{e\mu}^4 \left(s + m_h^2 N_h + \frac{m_h^3 - m_h \Gamma_{h^{(-)}}^2}{\Gamma_{h^{(-)}}} Q_h \right) \right) \qquad (54)$$

for the case of right polarized electrons, where

$$N_k = \ln \left| \frac{m_k^4 + \Gamma^2 m_k^2}{(s+m_k^2)^2 + \Gamma_k^2 m_k^2} \right|,$$
$$Q_k = \arctan\left(\frac{s+m_k^2}{\Gamma_k m_k} \right) - \arctan\left(\frac{m_k}{\Gamma_k} \right),$$

and $k = \tilde{\delta}, h$. In (53) and (54) the quantity λ_ν denotes the neutrino helicity. For the left polarized electrons and the right polarized neutrinos the deviations from the SM predictions are very small. For example, at $f_{e\mu} = 3 \times 10^{-2}$ they are of the order of 0.1%. Recall that in this case there is the W_1 resonance peak (Glashow resonance) and its height (σ_W) reaches values of the order of 10^4 pb.

From the definitions of m_{W_1} and M_D we could obtain

$$\alpha_{e\mu} = \frac{\Omega k_+}{k_2}. \qquad (55)$$

$$\sigma_{e^-\mu^-\to\Delta_1^{(--)}Z_1} = \frac{f_{e\mu}^2\beta_{\Delta_1 Z}^3 s^2}{4\pi m_{Z_1}^2}\left(\frac{\alpha_{\rm L}^2 c_{\theta_d}^4 P_+ + \alpha_{\rm R}^2 s_{\theta_d}^4 P_-}{(s-m_{\Delta_1}^2)^2+\Gamma_{\Delta_1}^2 m_{\Delta_1}^2}\right.$$
$$\left.+\frac{2c_{\theta_d}s_{\theta_d}(\alpha_{\rm R}^2 s_{\theta_d}^2 P_- - \alpha_{\rm L}^2 c_{\theta_d}^2 P_+)[(s-m_{\Delta_1}^2)(s-m_{\Delta_2}^2)+\Gamma_{\Delta_1}\Gamma_{\Delta_2}m_{\Delta_1}m_{\Delta_2}]}{[(s-m_{\Delta_1}^2)^2+\Gamma_{\Delta_1}^2 m_{\Delta_1}^2][(s-m_{\Delta_2}^2)^2+\Gamma_{\Delta_2}^2 m_{\Delta_2}^2]} + \frac{c_{\theta_d}^2 s_{\theta_d}^2(\alpha_{\rm L}^2 P_+ + \alpha_{\rm R}^2 P_-)}{(s-m_{\Delta_2}^2)^2+\Gamma_{\Delta_2}^2 m_{\Delta_2}^2}\right), \qquad (59)$$

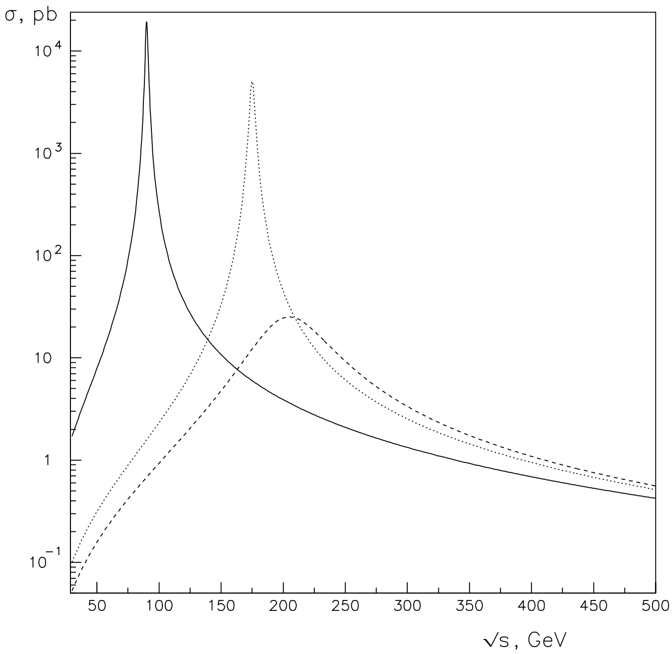

Fig. 5. The σ_{RR} versus $\sqrt{s}$ at $\lambda_\nu = -1$ for the cases **a** $m_h = 90\,\text{GeV}$ – solid line; **b** $m_h = 175\,\text{GeV}$ – dotted line; **c** $m_h = 200\,\text{GeV}$ – dashed line. In all three cases we adopt $\varphi_e = 8\times10^{-3}$, $\varphi_\mu = 9\times10^{-3}$, $f_{\tau\tau} = 0.5$ and $k_1 = 115\,\text{GeV}$

When M_D is non-zero, then even for large values of φ_e and φ_μ the quantity $\alpha_{e\mu}$ is very small. For example, at $\varphi_e = 2.5\times10^{-2}$, $\varphi_\mu = 3\times10^{-2}$ and $k_1 = 70\,\text{GeV}$, we have $\alpha_{e\mu} = 1.8\times10^{-5}$. Since for the definition of α_{ll} we have (51), the same is true for the case $l = e$, while the quantity $\alpha_{\mu\mu}$ could be as large as 10^{-2} even at small angles φ_μ. In the DBVEV case we have

$$\alpha_{ab} = 2h'_{ab}.$$

However, even at abnormally large values of $\alpha_{e\mu}$, say 2×10^{-2}, the cross section for the right polarized electrons and the left polarized electron neutrinos can reach only values of order 3×10^{-3} fb. Therefore, of primary interest are the cases with the initial $e_{\rm L}^-\nu_{e\rm L}$ and $e_{\rm R}^-\nu_{e\rm R}$. In Fig. 4 we represent the total cross section for the former case.

For small mixing angles φ_l, the value of $\sigma_{\rm R}$, at $\lambda_\nu = -1$, ($\sigma_{\rm RR}$) is very small. Figure 5 shows the behavior of $\sigma_{\rm RR}(s^{1/2})$ for the case of large mixing angles.

It is well to bear in mind that the following possibility could occur. The cross section of the singly charged Higgs boson resonances $(\sigma_{e^-\nu_e\to\mu^-\nu_\mu})_{\tilde\delta,h}$ turns out to be larger than σ_W. For example, when $f_{\tau\tau} = 0.6$, $f_{\mu\mu} = 0.75$, $f_{ee} = 0.09$ and $m_{\tilde\delta} = 70\,\text{GeV}$, we have $(\sigma_{e^-\nu_e\to\mu^-\nu_\mu})_{\tilde\delta} \sim 10^5$ pb.

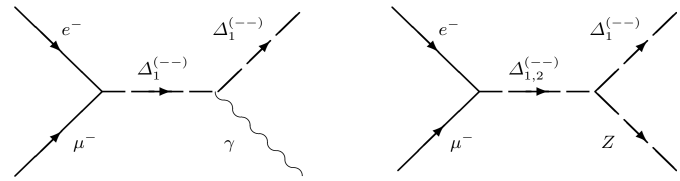

Fig. 6. The Feynman diagrams corresponding to the processes $e^-\mu^-\to\Delta_1^{(--)}\gamma$, $\quad e^-\mu^-\to\Delta_1^{(--)}Z_1$

At high energy the best way of measuring the non-diagonal YCs is to investigate the lepton flavor (LF) changing processes which has s channel resonance amplification or else is described by Feynman diagrams with only one vertex containing the YC. The processes

$$e^-\mu^-\to\Delta_i^{(--)}\gamma \qquad (i=1,2) \qquad (56)$$

$$e^-\mu^-\to\Delta_i^{(--)}Z_1, \qquad (57)$$

provide excellent examples. These could be studied at the muon colliders (MCs) under construction [28,29] which work both with a fixed electron target and with an electron beam. When $i = 1$, the Feynman diagrams corresponding to the processes (56) and (57) are displayed in Fig. 6.

Below we assume that the initial particles are unpolarized and we neglect the lepton masses. Then for the total cross sections we obtain the expressions (equation (59) stands on top of the page)

$$\sigma_{e^-\mu^-\to\Delta_1^{(--)}\gamma} = \frac{2e^2 f_{e\mu}^2(c_{\theta_d}^2 P_+ + s_{\theta_d}^2 P_-)(s^2-m_{\Delta_1}^4)}{\pi s[(s-m_{\Delta_1}^2)^2+\Gamma_{\Delta_1}^2 m_{\Delta_1}^2]}, \qquad (58)$$

where $P_\pm = (1\pm\lambda_e)(1\pm\lambda_\mu)$, and λ_l denotes the helicity of the initial leptons.

The quantity $\sigma_{e^-\mu^-\to\Delta_1^{(--)}\gamma}$ is maximum at the threshold $(s^{1/2})_{th}$. However, there are two factors constraining our possibilities when we investigate the behavior of the reaction (56) near $(s^{1/2})_{\rm th}$.

(i) By virtue of the fact that the $\Delta_1^{(--)}$ boson is an unstable particle $(s^{1/2})_{\rm th}$ is smeared within the region of energy being of order $\Delta E_1 = \Gamma_{\Delta_1\to all}$.
(ii) Taking into account the radiation corrections and the contributions coming from the soft photon bremsstrahlung leads not only to the cancellation of the infrared divergence, but to a dependence of the cross section on the photon resolution of the detector (ΔE_γ) which is used in the experiments.

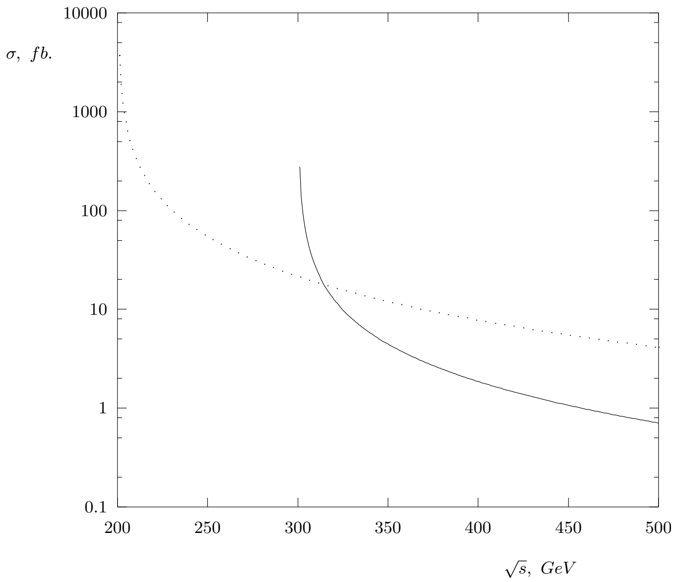

Fig. 7. The total cross section of the reaction $e^-\mu^- \to \Delta_1^{(--)}\gamma$ versus $\sqrt{s}$ at $\lambda_e = 0.8$, $\lambda_\mu = 0.9$ and $\Delta E_\gamma = 1\,\mathrm{GeV}$. The solid (dotted) line corresponds to the case $m_{\Delta_1} = 200$ ($m_{\Delta_1} = 300$) GeV and $f_{e\mu} = 3\times 10^{-3}$ ($f_{e\mu} = 10^{-4}$)

The total decay width of the Δ_1 boson is much smaller than ΔE_γ. At $f_{\tau\tau} = 0.9$ and $m_{\Delta_1} = 400\,\mathrm{GeV}$ it is a mere 0.25 MeV, while, for example, the photon energy threshold of the electromagnetic calorimeter of the OPAL detector is about 1 GeV. In Fig. 7 we display the behavior of $\sigma_{e^-\mu^-\to\Delta_1^{(--)}\gamma}$ in the energy region from $(s^{1/2})_{\mathrm{th}} + \Delta E_\gamma$ to 500 GeV.

Since the reaction (56) has practically no background, it is evident that its investigation is one of the most precise methods to measure the value of $f_{e\mu}m_{\Delta_1}^{-2}$.

Investigating the reaction (57) at $s^{1/2} = m_{\Delta_j}$ $(j \neq i)$ we obtain information about the masses and the mixing angle of the $\Delta_1^{(--)}$ and $\Delta_2^{(--)}$ bosons. We recall that the MC is an ideal tool for such research because of the following reasons. First of all, besides the storage ring of the MC one has a special-purpose ring which allows to optimize the collider luminosity near the Higgs resonance under study. Second, the root mean square deviation of the beam energy from the Gaussian distribution could sharpen to 0.01%, which in its turn allows one to reach the necessary condition to measure even the small decay widths of the doubly charged Higgs bosons $\Gamma_{\Delta_i\to all}$ (the beam energy resolution must be of the same order as the quantity $\Gamma_{\Delta_i\to all}$).

For the sake of definiteness we assume that $m_{\Delta_1} < m_{\Delta_2}$. From the absence of the $\Delta_2^{(--)}$ resonance peak in the cross section we must conclude that one of the following conditions is fulfilled:

$$m_{\Delta_1} + m_{Z_1} > m_{\Delta_2}, \qquad \theta_d \text{ is arbitrary}, \tag{60}$$

$$m_{\Delta_1} + m_{Z_1} \leq m_{\Delta_2} \qquad \theta_d \approx 0. \tag{61}$$

In Fig. 8 we show $\sigma_{e^-\mu^-\to\Delta_1^{(--)}Z_1}$ versus $s^{1/2}$ for the more optimistic case when $m_{\Delta_1} + m_{Z_1} < m_{\Delta_2}$ and $\theta_d \neq 0$.

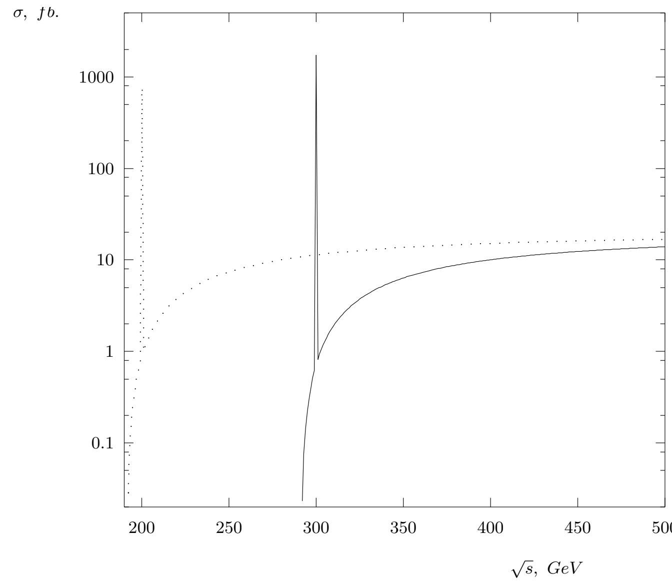

Fig. 8. The total cross section of the reaction $e^-\mu^- \to \Delta_1^{(--)}Z_1$ versus $\sqrt{s}$ at $\lambda_e = 0.8$, $\lambda_\mu = 0.9$, $\theta_d = 10^{-4}$ and $f_{e\mu} = 3\times 10^{-3}$. The solid (dotted) line corresponds to the case $m_{\Delta_1} = 200\,\mathrm{GeV}$, $m_{\Delta_2} = 300\,\mathrm{GeV}$ ($m_{\Delta_1} = 100\,\mathrm{GeV}$, $m_{\Delta_2} = 200\,\mathrm{GeV}$)

From now on we shall consider the process of the electron–muon recharge,

$$e^-\mu^+ \longrightarrow e^+\mu^-, \tag{62}$$

which practically has no background. One can start to investigate this process right now, since the energy of the muon beams used in the current experiments is rather high. Thus, for example, since 1994 the Spin Muon Collaboration at CERN has been working with muon beams of which the energy reaches 190 GeV and the polarization ~ 0.8 [30]; in the FNAL experiments investigating the muon–proton interaction muons with energies of 470 GeV [31] have been used. The process (62) could be also studied at the MCs. In the second order of perturbation theory its diagrams are given in Fig. 9. We recall, that in the two Higgs doublet model, provided the YCs have LF non-diagonal elements (see, for example, [32]), the process of the electron–muon recharge is also described by diagrams with neutral Higgs boson exchanges. On the other hand, in the case of the SM this process takes place only in the fourth order of perturbation theory (provided the neutrino is massive) and there are no diagrams with a physical Higgs boson in the virtual states.

For the sake of simplicity we set

$$h_{e\mu} = h_{\mu e}, \tag{63}$$

and set θ_d to zero. Note that from (63) follows

$$\varphi_e = \varphi_\mu.$$

We shall also assume that the Yukawa Lagrangian for the quarks is defined by (14).

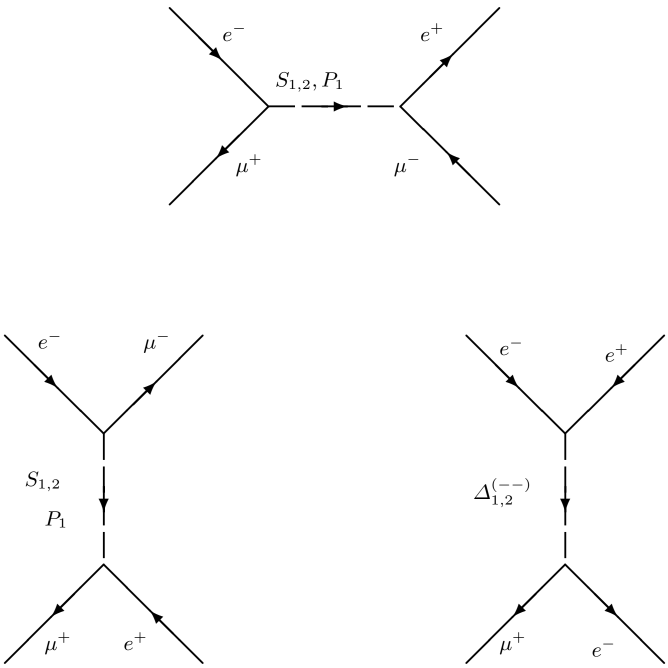

Fig. 9. The Feynman diagrams corresponding to the process of the electron-muon recharge

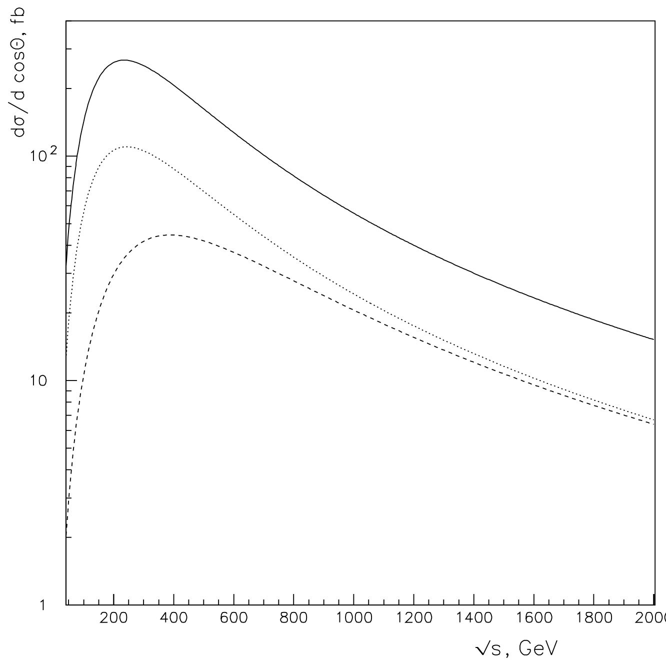

Fig. 10. The differential cross section of the process $e^-\mu^+ \to e^+\mu^-$ versus $\sqrt{s}$ at $\theta = \pi/2$, $\varphi_e = \varphi_\mu = 10^{-4}$, $f_{ee} = 0.15$, and $f_{\mu\mu} = 0.3$. The solid line corresponds to $\lambda_e = \lambda_\mu = 0$, $m_{\Delta_1} = 150\,\mathrm{GeV}$, $m_{\Delta_2} = 200\,\mathrm{GeV}$, the dotted line does to $m_{\Delta_1} = 150\,\mathrm{GeV}$, $m_{\Delta_2} = 200\,\mathrm{GeV}$, $\lambda_e = 0.7$, and $\lambda_\mu = -0.8$, the dashed line does to $m_{\Delta_1} = 250\,\mathrm{GeV}$, $m_{\Delta_2} = 300\,\mathrm{GeV}$, $\lambda_e = 0.7$, and $\lambda_\mu = -0.8$

The differential cross section of the reaction (62) for the initial polarized particles is defined by

$$
\begin{aligned}
\frac{\mathrm{d}\sigma}{\mathrm{d}(\cos\theta)} = \frac{1}{32\pi s}\Bigg\{&(1+\lambda_e)(1+\lambda_\mu)\frac{4(f_{\mu\mu}f_{ee})^2t^2}{[(t-m_{\Delta_1}^2)^2+\Gamma_{\Delta_1}^2 m_{\Delta_1}^2}\\
&+(1\to 2, \lambda_e\to-\lambda_e, \lambda_\mu\to\lambda_\mu)\\
&+s^2(1-\lambda_e\lambda_\mu)\\
&\times\Bigg[\Bigg|\frac{(h_{e\mu}k_1-h'_{e\mu}k_2)^2 s_{\theta_0}^2}{k_+^2(s-m_{S_1}^2+\mathrm{i}\Gamma_{S_1}m_{S_1})}\\
&+\frac{(h_{e\mu}k_1-h'_{e\mu}k_2)^2 c_{\theta_0}^2}{k_+^2(s-m_{S_2}^2+\mathrm{i}\Gamma_{S_2}m_{S_2})}\Bigg|^2\\
&+\frac{(h_{e\mu}k_1+h'_{e\mu}k_2)^4}{k_+^4[(s-m_{P_1}^2)^2+\Gamma_{P_1}^2 m_{P_1}^2]}\Bigg]\\
&+(s\to u, \lambda_e=\lambda_\mu=0)\\
&+su(1-\lambda_e\lambda_\mu)\sum_{k,l=P_1,S_{1,2}}F_{kl}\\
&+2u^2\lambda_e\lambda_\mu\sum_{i=1,2}G_{P_1S_i}\Bigg\},
\end{aligned}
\quad (64)
$$

where

$$
\begin{aligned}
F_{kl} &= \frac{a_{kl}[(u-m_k^2)(s-m_l^2)+m_k\Gamma_k m_l\Gamma_l]}{[(u-m_k^2)^2+m_k^2\Gamma_k^2][(s-m_l^2)^2+m_l^2\Gamma_l^2]},\\
G_{P_1S_i} &= \frac{a_{P_1S_i}[(u-m_{S_i}^2)(u-m_{P_1}^2)+m_{S_i}\Gamma_{S_i}m_{P_1}\Gamma_{P_1}]}{[(u-m_{S_i}^2)^2+m_{S_i}^2\Gamma_{S_i}^2][(u-m_{P_1}^2)^2+m_{P_1}^2\Gamma_{P_1}^2]},\\
a_{S_1S_1} &= \frac{(h_{e\mu}k_1-h'_{e\mu}k_2)^4 s_{\theta_0}^4}{k_+^4},
\end{aligned}
$$

$$
\begin{aligned}
a_{S_2S_2} &= a_{S_1S_1}\,(\theta_0\to\theta_0+\pi/2),\\
a_{P_1P_1} &= \frac{(h_{e\mu}k_1+h'_{e\mu}k_2)^4}{k_+^4},\\
a_{P_1S_2} &= a_{S_2P_1} = -\frac{(h_{e\mu}k_1-h'_{e\mu}k_2)^2(h_{e\mu}k_1+h'_{e\mu}k_2)^2 c_{\theta_0}^2}{k_+^4},\\
a_{P_1S_1} &= a_{S_1P_1} = a_{S_2P_1}\,(\theta_0\to\theta_0+\pi/2),\\
a_{S_1S_2} &= a_{S_2S_1}\\
&= \frac{(h_{e\mu}k_1-h'_{e\mu}k_2)^2(h_{e\mu}k_1+h'_{e\mu}k_2)^2 c_{\theta_0}^2 s_{\theta_0}^2}{k_+^4};
\end{aligned}
$$

θ is the angle between $\vec{p_e}$ and $\vec{p_\mu}$ in the center of mass system, and the expressions for the decay widths of the $\Delta_{1,2}^{(--)}$, $S_{1,2}$ and P_1 bosons are given in the Appendix.

As the analysis shows, the cross section of the reaction (62) also depends on k_1. It increases while k_1 is approaching k_g and decreases while moving away from k_g both to the region of smaller values as well as larger ones.

Let us choose the following values for the LRM parameters entering the cross section of the process (62)

$$
m_{S_2} = 900\,\mathrm{GeV}, \qquad g_\mathrm{R} = 1.4 g_\mathrm{L}, \qquad m_{P_1} = 5\,\mathrm{TeV},
$$

$$
m_{N_e} = 750\,\mathrm{GeV}, \qquad m_{N_\mu} = 1.8\,\mathrm{TeV}.
$$

In Fig. 10 we present the differential cross section of the reaction (62) for small mixing angles φ_l and a large triplet YCs f_{ll}. In this case we have no chance to detect the

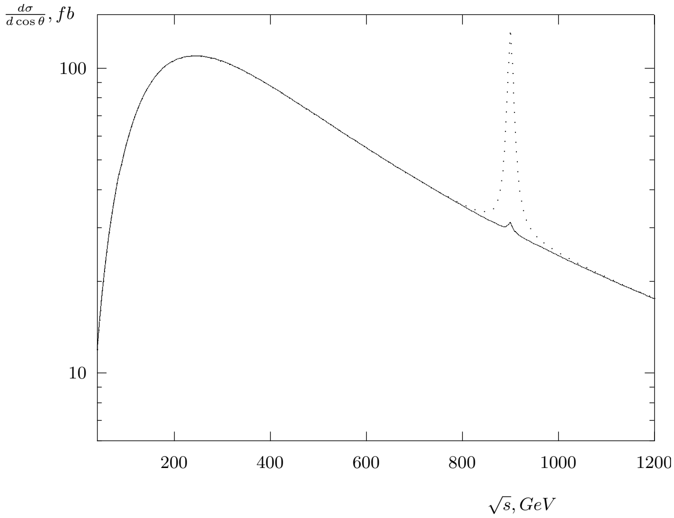

Fig. 11. The differential cross section of the process $e^-\mu^+ \to e^+\mu^-$ versus $\sqrt{s}$ at $\theta = \pi/2$. The solid line corresponds to $\varphi_e = 10^{-2}$, $\varphi_\mu = 2\times10^{-2}$, $m_{S_1} = 85\,\mathrm{GeV}$, and $k_1 = 115\,\mathrm{GeV}$. The dashed line corresponds to the case of the DBVEV at $h_{e\mu} = 3\times10^{-2}$. For the both curves the following values of the parameters. $m_{\Delta_1} = 150\,GeV$, $m_{\Delta_2} = 200\,GeV$, $\lambda_e = 0.7$, $\lambda_\mu = -0.8$, $m_{S_1} = 90\,GeV$ have been used

resonance peaks connected with the Higgs boson $S_{1,2}$ and P_1. As follows from Fig. 10 the cross section decreases with increasing doubly charged Higgs bosons masses.

In the case of large mixing angles φ_l ($\varphi_\mu = 2\times10^{-2}$) the partial cross sections associated with the neutral Higgs bosons exchanges are very small. For example, when $k_1 = 118\,\mathrm{GeV}$ and $\theta = \pi/2$, the height of the S_1 resonance peak is equal to 0.3 and 0.17 fb for $m_{S_1} = 65$ and $90\,\mathrm{GeV}$, respectively.

In Fig. 11 we display $\mathrm{d}\sigma/\mathrm{d}(\cos\theta)$ versus $s^{1/2}$ both for large mixing angles and for the DBVEV (in the case of the DBVEV the S_1 and P_1 Higgs bosons do not take part in the process (62) at all). Under these conditions just one S_2 resonance could be observed. At increasing f_{ee} and $f_{\mu\mu}$ the contributions coming from the diagrams with the $\Delta_{1,2}^{(--)}$ exchanges are growing, which in turn might lead to the absence of the S_2 resonance splash in the cross section even in the DBVEV case with the large values of the bidoublet YCs $h_{e\mu}$ and $h'_{e\mu}$. Thus, for example at $f_{ee} = 0.42$, $f_{\mu\mu} = 0.65$, $m_{\Delta_1} = 150$, $m_{\Delta_2} = 200\,\mathrm{GeV}$ and $h_{e\mu} = 3\times10^{-2}$ in the unpolarized cross section the S_2 resonance peak is no longer observed. It should be stressed that the registration of the S_2 resonance peak in the reaction (62) will also ascertain the existence of the $\nu_e \longleftrightarrow \nu_\mu$ oscillations.

4 Conclusions

In this paper we have studied the Higgs sector of the asymmetric LRM. We have chosen the LRM as an example not by accident but due to the reason that its Higgs sector contains the elements belonging to other nowaday most popular models. Thus, for example, the presence of the bidoublet in the LRM causes the existence of the same physical Higgs bosons in the minimal supersymmetric standard model and in the two Higgs doublet modification of the SM.

We have considered two reactions,

$$e^-\nu_e \to W_1^- Z_1, \qquad e^-\nu_e \to \mu^-\nu_\mu.$$

For the former within the SM any resonances are absent while the LRM predicts the existence of the $\tilde{\delta}^{(-)}$ and $h^{(-)}$ resonances, whose heights could reach values of the order of a few $\times10^3$ pb. For the latter, at $s^{1/2} = m_W$, there is the so-called Glashow resonance predicted both by the SM and the LRM. However, the LRM total cross section also has two resonance peaks related to the $\tilde{\delta}^{(-)}$ and $h^{(-)}$ Higgs bosons. At the corresponding values of the YCs and the Higgs potential parameters the height of these peaks could exceed that of the Glashow resonance. The UHE cosmic neutrino could be used for studying these two reactions at such neutrino telescopes as DUMAND, BAIKAL NT-200, NESTOR and AMANDA. Then the observation of heavy muon showers connected with the resonance splashes in the cross sections will definitely ascertain both the existence of the singly charged Higgs bosons and the character of their interaction with quarks and leptons.

We have shown that the process

$$e^-\mu^- \to \Delta_1^{(--)}\gamma$$

is the ideal tool for measuring the quantity $f_{e\mu}m_{\Delta_1}^{-2}$. It should be stressed that up to now we have only bounds on the combination $f_{e\mu}f_{ee}m_{\Delta_1}^{-2}$.

It was found that with the help of the process

$$e^-\mu^- \to \Delta_1^{(--)}Z_1,$$

utilizing the exchange of the $\Delta_2^{(--)}$ boson in the s channel, we could obtain bounds on both the masses and the mixing angle of the $\Delta_1^{(--)}$ and $\Delta_2^{(--)}$ bosons. Note that the $\Delta_2^{(--)}$ resonance peak height crucially depends on the value of the above mentioned angle, which in its turn is extremely sensitive to the choice of both the VEV and the Higgs potential parameters.

We have considered the reaction of the electron–muon recharge

$$e^-\mu^+ \to e^+\mu^-,$$

which gives information about such YCs as $f_{\mu\mu}$, f_{ee}, $h_{e\mu}$ and $h'_{e\mu}$. It was shown that under the DBVEV when the bidoublet YCs could be large, there is a good chance to detect the neutral Higgs boson S_2 by a resonance splash in the cross section.

We have also obtained the relationships linking the YCs with the masses and the mixing angles of the neutrinos. Using the YCs values found in the accelerator experiment with the Higgs bosons we get information about the neutrino sector parameters. Therefore, the program aiming at looking for the physical Higgs bosons at the lepton colliders will also provide the answer to the question of the masses and the oscillation angles of the neutrinos. In this sense the lepton colliders shall turn out to be complementary to neutrino telescopes.

Appendix

In [5] it was shown that the most general form of the Higgs potential of the LRM is

$$\begin{aligned}
V = &-\mu_1^2[\mathrm{Sp}(\Phi^\dagger\Phi)] - \mu_2^2[\mathrm{Sp}(\tilde{\Phi}\Phi^\dagger) + \mathrm{Sp}(\tilde{\Phi}^\dagger\Phi)] \\
&-\mu_3^2[\mathrm{Sp}(\Delta_\mathrm{L}\Delta_\mathrm{L}^\dagger) + \mathrm{Sp}(\Delta_\mathrm{R}\Delta_\mathrm{R}^\dagger)] + \lambda_1[\mathrm{Sp}(\Phi\Phi^\dagger)]^2 \\
&+\lambda_2\{[\mathrm{Sp}(\tilde{\Phi}\Phi^\dagger)]^2 + [\mathrm{Sp}(\tilde{\Phi}^\dagger\Phi)]^2\} \\
&+\lambda_3[\mathrm{Sp}(\tilde{\Phi}\Phi^\dagger)\mathrm{Sp}(\tilde{\Phi}^\dagger\Phi)] \\
&+\lambda_4\{\mathrm{Sp}(\Phi\Phi^\dagger)[\mathrm{Sp}(\tilde{\Phi}\Phi^\dagger) + \mathrm{Sp}(\tilde{\Phi}^\dagger\Phi)]\} \\
&+\rho_1\{[\mathrm{Sp}(\Delta_\mathrm{L}\Delta_\mathrm{L}^\dagger)]^2 + [\mathrm{Sp}(\Delta_\mathrm{R}\Delta_\mathrm{R}^\dagger)]^2\} \\
&+\rho_2[\mathrm{Sp}(\Delta_\mathrm{L}\Delta_\mathrm{L})\mathrm{Sp}(\Delta_\mathrm{L}^\dagger\Delta_\mathrm{L}^\dagger) + \mathrm{Sp}(\Delta_\mathrm{R}\Delta_\mathrm{R})\mathrm{Sp}(\Delta_\mathrm{R}^\dagger\Delta_\mathrm{R}^\dagger)] \\
&+\rho_3[\mathrm{Sp}(\Delta_\mathrm{L}\Delta_\mathrm{L}^\dagger)\mathrm{Sp}(\Delta_\mathrm{R}\Delta_\mathrm{R}^\dagger)] \\
&+\rho_4[\mathrm{Sp}(\Delta_\mathrm{L}\Delta_\mathrm{L})\mathrm{Sp}(\Delta_\mathrm{R}^\dagger\Delta_\mathrm{R}^\dagger) + \mathrm{Sp}(\Delta_\mathrm{L}^\dagger\Delta_\mathrm{L}^\dagger)\mathrm{Sp}(\Delta_\mathrm{R}\Delta_\mathrm{R})] \\
&+\alpha_1\{\mathrm{Sp}(\Phi\Phi^\dagger)[\mathrm{Sp}(\Delta_\mathrm{L}\Delta_\mathrm{L}^\dagger) + \mathrm{Sp}(\Delta_\mathrm{R}\Delta_\mathrm{R}^\dagger)]\} \\
&+\alpha_2[\mathrm{Sp}(\Phi\tilde{\Phi}^\dagger)\mathrm{Sp}(\Delta_\mathrm{R}\Delta_\mathrm{R}^\dagger) + \mathrm{Sp}(\Phi^\dagger\tilde{\Phi})\mathrm{Sp}(\Delta_\mathrm{L}\Delta_\mathrm{L}^\dagger)] \\
&+\alpha_2^*[\mathrm{Sp}(\Phi^\dagger\tilde{\Phi})\mathrm{Sp}(\Delta_\mathrm{R}\Delta_\mathrm{R}^\dagger) + \mathrm{Sp}(\tilde{\Phi}^\dagger\Phi)\mathrm{Sp}(\Delta_\mathrm{L}\Delta_\mathrm{L}^\dagger)] \\
&+\alpha_3[\mathrm{Sp}(\Phi\Phi^\dagger\Delta_\mathrm{L}\Delta_\mathrm{L}^\dagger) + \mathrm{Sp}(\Phi^\dagger\Phi\Delta_\mathrm{R}\Delta_\mathrm{R}^\dagger)] \\
&+\beta_1[\mathrm{Sp}(\Phi\Delta_\mathrm{R}\Phi^\dagger\Delta_\mathrm{L}^\dagger) + \mathrm{Sp}(\Phi^\dagger\Delta_\mathrm{L}\Phi\Delta_\mathrm{R}^\dagger)] \\
&+\beta_2[\mathrm{Sp}(\tilde{\Phi}\Delta_\mathrm{R}\Phi^\dagger\Delta_\mathrm{L}^\dagger) + \mathrm{Sp}(\tilde{\Phi}^\dagger\Delta_\mathrm{L}\Phi\Delta_\mathrm{R}^\dagger)] \\
&+\beta_3[\mathrm{Sp}(\Phi\Delta_\mathrm{R}\tilde{\Phi}^\dagger\Delta_\mathrm{L}^\dagger) + \mathrm{Sp}(\Phi^\dagger\Delta_\mathrm{L}\tilde{\Phi}\Delta_\mathrm{R}^\dagger)]. \qquad \text{(A.1)}
\end{aligned}$$

Below we provide the expressions defining the total decays widths of the physical Higgs bosons. For the neutral scalar Higgs bosons they have the form

$$\Gamma_{S_i} = \Gamma_{S_i\to\mathrm{fermions}} + \sum_{nl}\Gamma_{S_i\to W_nW_l},$$

where

$$\begin{aligned}
\Gamma_{S_1\to\mathrm{leptons}} = &\frac{1}{16\pi m_{S_1}}\Bigg\{\sum_a\left(\frac{m_a c_{\theta_0}}{k_+}\right)^2\mathcal{G}(S_1,l_a,l_a) \\
&+\sum_{a,b}\frac{(h_{ab}k_1 - h'_{ab}k_2)^2 s_{\theta_0}^2}{k_+^2}\mathcal{G}(S_1,l_a,l_b) \\
&+\sum_{a,b}\frac{[h_{ab}(k_1c_{\theta_0} - k_2s_{\theta_0}) + h'_{ab}(k_1s_{\theta_0} + k_2c_{\theta_0})]^2}{k_+^2} \\
&\times\mathcal{G}(S_1,\nu_a,N_b)\Bigg\}, \qquad \text{(A.2)}
\end{aligned}$$

$$\mathcal{G}(S_i,f_j,f_k) = \left(1 - \frac{(m_j+m_k)^2}{m_{S_i}^2}\right)w(m_{S_i}^2,m_j^2,m_k^2),$$

$$w(a,b,c) = [a^2+b^2+c^2-2(ab+ac+bc)]^{1/2}$$

$$\begin{aligned}
\Gamma_{S_1\to\mathrm{quarks}} = &\frac{1}{16\pi m_{S_1}} \\
&\times\Bigg\{\sum_a\frac{[m_{u_a}(c_{\theta_0}k_2 + s_{\theta_0}k_1)]^2}{(k_2k_+)^2}\mathcal{G}(S_1,u_a,u_a) \\
&+\sum_a\frac{[m_{d_a}(c_{\theta_0}k_1 - s_{\theta_0}k_2)]^2}{(k_1k_+)^2}\mathcal{G}(S_1,d_a,d_a)\Bigg\},
\end{aligned}$$

$$\Gamma_{S_2\to\mathrm{fermions}} = \Gamma_{S_1\to fermions}(\theta_0\to\theta_0+\pi/2, m_{S_1}\to m_{S_2}),$$

$$\begin{aligned}
\Gamma_{S_i\to W_nW_l} = &\frac{(c_i^{nl})^2}{16\pi m_{S_i}^3}\left(2 + \frac{(m_{S_i}^2 - m_{W_n}^2 - m_{W_l}^2)^2}{4m_{W_n}^2m_{W_l}^2}\right) \\
&\times w(m_{S_i}^2,m_{W_n}^2,m_{W_l}^2),
\end{aligned}$$

$$2c_1^{nl} = \begin{cases}
k_+c_{\theta_0}(g_\mathrm{L}^2c_\xi^2 + g_\mathrm{R}^2s_\xi^2) & \\
+g_\mathrm{L}g_\mathrm{R}s_{2\xi}(2k_1k_2c_{\theta_0} + k_-^2s_{\theta_0})/k_+, & n=l=1 \\
k_+c_{\theta_0}(g_\mathrm{L}^2s_\xi^2 + g_\mathrm{R}^2c_\xi^2) & \\
-g_\mathrm{L}g_\mathrm{R}s_{2\xi}(2k_1k_2c_{\theta_0} + k_-^2s_{\theta_0})/k_+, & n=l=2 \\
k_+c_{\theta_0}(g_\mathrm{L}^2 - g_\mathrm{R}^2)s_{2\xi}/2 & \\
-g_\mathrm{L}g_\mathrm{R}c_{2\xi}(2k_1k_2c_{\theta_0} + k_-^2s_{\theta_0})/k_+, & n=1, l=2
\end{cases}$$

$$c_2^{nl} = c_1^{nl}(\theta_0\to\theta_0+\pi/2).$$

For the neutral pseudoscalar Higgs bosons these widths are given by

$$\Gamma_{P_1} = \Gamma_{P_1\to\mathrm{fermions}} + 2\Gamma_{P_1\to W_1^+W_2^-},$$

where

$$\begin{aligned}
\Gamma_{P_1\to\mathrm{leptons}} = &\frac{1}{16\pi m_{P_1}}\Bigg\{\sum_{a,b}\frac{(h_{ab}k_1 + h'_{ab}k_2)^2}{k_+^2}\mathcal{G}'(P_1,l_a,l_b) \\
&+\frac{(h_{ab}k_2 + h'_{ab}k_1)^2}{k_+^2}\mathcal{G}'(P_1,\nu_a,N_b)\Bigg\},
\end{aligned}$$

$$\begin{aligned}
\Gamma_{P_1\to\mathrm{quarks}} = &\frac{1}{16\pi m_{P_1}}\sum_a\Bigg\{\left(\frac{m_{u_a}k_1}{k_2k_+}\right)^2\mathcal{G}'(P_1,u_a,u_a) \\
&+\left(\frac{m_{d_a}k_2}{k_1k_+}\right)^2\mathcal{G}'(P_1,d_a,d_a)\Bigg\},
\end{aligned}$$

$$\mathcal{G}'(S_i,f_j,f_k) = \mathcal{G}(S_i,f_j,f_k)(m_j\to -m_j),$$

$$\begin{aligned}
\Gamma_{P_1\to W_1^+W_2^-} = &\frac{(g_\mathrm{L}g_\mathrm{R}k_+)^2}{32\pi m_{P_1}^3}\left(2 + \frac{(s - m_{W_1}^2 - m_{W_2}^2)^2}{4m_{W_1}^2m_{W_2}^2}\right) \\
&\times w(m_{P_1}^2,m_{W_1}^2,m_{W_2}^2).
\end{aligned}$$

Finally, the decay widths both of the singly charged and the doubly charged Higgs bosons are determined by

$$\Gamma_{\tilde{\delta}} = \sum_l(\Gamma_{\tilde{\delta}\to l^c\nu_l} + \Gamma_{\tilde{\delta}\to l^cN_l}) + \Gamma_{\tilde{\delta}\to W_1Z_1}, \qquad \text{(A.3)}$$

where

$$\Gamma_{\tilde{\delta}\to l^c\nu_l} = \frac{f_{ll}^2d^2(m_{\tilde{\delta}}^2 - m_l^2 - m_{\nu_l}^2)}{8\pi m_{\tilde{\delta}}^3}w(m_{\tilde{\delta}}^2,m_l^2,m_{\nu_l}^2),$$

$$\Gamma_{\tilde{\delta}\to l^cN_l} = \frac{f_{ll}^2a^2\beta^2k_0^2(m_{\tilde{\delta}}^2 - m_l^2 - m_{N_l}^2)}{8\pi m_{\tilde{\delta}}^3(\alpha - \rho_1 - \rho_3/2)^2v_\mathrm{R}^2}w(m_{\tilde{\delta}}^2,m_l^2,m_{N_l}^2),$$

$$\begin{aligned}
\Gamma_{\tilde{\delta}\to W_1Z_1} = &\frac{(g_\mathrm{R}g_\mathrm{L}s_W^2c_\Phi s_\xi a\beta k_0)^2}{16\pi c_W^2m_{\tilde{\delta}}^3(\alpha + \rho_1 - \rho_3/2)^2} \\
&\times\left(\frac{(m_{\tilde{\delta}}^2 - m_{W_1}^2 - m_{Z_1}^2)^2}{4m_{W_1}^2m_{Z_1}^2} + 2\right) \\
&\times w(m_{\tilde{\delta}}^2,m_{W_1}^2,m_{Z_1}^2).
\end{aligned}$$

$$\Gamma_{h\to l\overline{\nu}_l} = \Gamma_{\tilde{\delta}\to l^c\nu_l} \times \left(\sqrt{2}f_{ll}d \to \frac{b(h'_{ll}k_2 - h_{ll}k_1)}{k_+},\ m_{\tilde{\delta}} \to m_h\right),$$

$$\Gamma_{h\to l\overline{N}_l} = \Gamma_{\tilde{\delta}\to l^c N_l} \times \left(\frac{ak_0}{v_\mathrm{R}} \to \frac{b(h_{ll}k_2 - h'_{ll}k_1)}{k_+},\ m_{\tilde{\delta}} \to m_h\right),$$

$$\Gamma_{h\to W_1Z_1} = \Gamma_{\tilde{\delta}\to W_1Z_1} \left(\frac{g_\mathrm{R}g_\mathrm{L}s_W^2 c_\Phi s_\xi a\beta k_0}{c_W(\alpha+\rho_1-\rho_3/2)} \to \frac{g_\mathrm{R}g_\mathrm{L}c_\Phi s_\xi b k_-^2}{\sqrt{2}k_+ c_W},\ m_{\tilde{\delta}} \to m_h\right),$$

$$\Gamma_{\Delta_{1,2}} \simeq \frac{f_{\tau\tau}^2 m_\tau^2}{8\pi m_{\Delta_{1,2}}}\sqrt{1-\frac{4m_\tau^2}{m_{\Delta_{1,2}}^2}}.$$

References

1. H.P. Nilles, Phys. Rep. **110**, 1 (1984); H. Haber, G. Kane, Phys. Rep. **117**, 75 (1985)
2. J.C. Pati, A. Salam, Phys. Rev. D **10**, 275 (1974); R.N. Mohapatra, J.C. Pati, Phys. Rev. D **11**, 566, 2558 (1975); G. Senjanovich, R.N. Mohapatra, Phys. Rev. D **12**, 1502 (1975)
3. R.N. Mohapatra, Progr. Part. Nucl. Phys. **26**, 165 (1991)
4. O.M. Boyarkin, Acta Phys. Polonica B **23**, 1031 (1992); O.M. Boyarkin, Phys. Rev. D **50**, 2247 (1994)
5. N.G. Deshpande et al., Phys. Rev. D **44**, 837 (1991)
6. Review of Particle Physics, Eur. Phys. J. C **3**, 1–794 (1998)
7. R.N. Mohapatra, Fortschr. Phys., **31**, 185 (1983)
8. R.N. Francis et al., Phys. Rev. D **43**, 2369 (1991)
9. B.T. Cleveland et al., Nucl. Phys. (Proc. Suppl.) B **38**, 47 (1995); GALLEX Collaboration, W. Hampel et al., Phys. Lett. B **388**, 384 (1996); SAGE Collaboration, J.N. Abdurashitov et al., Phys. Rev. Lett. **77**, 4708 (1996); Super-Kamiokande Collaboration, talk by Y. Suzuki at Neutrino-98, Takayama, Japan, June 1998; J.N. Bahcall, S. Basu, M.N. Pinsonneault, Phys. Lett. B **443**, 1 (1998)
10. L. Wolfenstein, Phys. Rev. D **17**, 20 (1978); S.P. Mikheyev, A.Yu. Smirnov, Yad. Fiz., **42**, 1441 (1985); X. Shi, D.N. Schramm, J.N. Bahcall, Phys. Rev. Lett., **69**, 717 (1992); N. Hata, P. Langacker, Phys. Rev. D **52**, 420 (1995)
11. R.Beccker-Szendy et al., Phys. Rev. D **46**, 3720 (1992)
12. W.W.M. Allison, Phys. Lett. B **391**, 491 (1997)
13. M. Apollonio et al., CHOOZ Collaboration, Phys. Lett. B **420**, 397 (1998)
14. G.L. Fogli, E. Lisi, A. Marrone, G. Scioscia, hep-ph/9808205
15. C. Athanassopoulos et al., Phys. Rev. Lett. **77**, 3082 (1996)
16. K. Eitel et al., Nucl. Phys. Proc. Suppl. **70**, 210 (1999)
17. The K2K Collaboration, Y. Oyama, Proceedings of the YITP Workshop on Flavor Physics, Kyoto, Japan 1998, hep-ex/9803014
18. The MINOS Collaboration, Neutrino oscillation physics at Fermilab: The NuMI-MINOS Project, Report NUMI-L-375, May 1998
19. P. Picchi, F. Pietropaolo, hep-ph/9812222
20. G.G. Boyarkina, O.M. Boyarkin, Yad. Fiz. **60**, 683 (1997); S. Geer, Phys. Rev. D **57**, 6989 (1998)
21. O.M. Boyarkin, Phys. Rev. D **53**, 5298 (1996)
22. R. Mohapatra, Phys. Rev. D **46**, 2990 (1992)
23. D.A. Moris, A. Ringwald, Preprint DESY 94-026, UCLA/94/TEP/9, 1994
24. R. Brown et al., Phys. Rev. **20**, 1164 (1979)
25. R.N. Mohapatra, D. Sidhu, Phys. Rev. Lett. **38**, 667 (1977)
26. O.M. Boyarkin, Yader. Fiz. **54**, 839 (1991) [Sov.J. Nucl. Phys. **54**, 506 (1991)]
27. G. Alexsander et al., Phys. Lett. B **370**, 174 (1996)
28. Proceedings of the Second Workshop on the Physics Potential and Development of $\mu^+\mu^-$ Colliders, Sausalito, California. 1994. Nucl. Instrum. and Meth. A **350**, 24 (1994)
29. Proceedings of the 9th Advanced ICFA beam dynamics Workshop: Beam dynamics and technology issues for $\mu^+\mu^-$ colliders, Montauk, Long Island 1995. To be published
30. SMC, D. Adams et al., Phys. Lett. B **357**, 248 (1995)
31. M.R. Adams et al., The E 665 Collaboration, FERMILAB-PUB-97/103-E94, E665, 1997
32. D. Atwood, L. Reina, A. Soni, Phys. Rev. D **55**, 317 (1997)